

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod is at rest in limiting equilibrium.

- (a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A . **Give a reason for your answer.**

(1)

(a) Friction at A must be acting to the right, towards the wall, because the only other horizontal force is the Normal Reaction from the wall, and these forces need to balance for horizontal equilibrium.

In an initial model, the rod is modelled as being **uniform**.

(b) By taking moments about A , show that

$$S = \frac{1}{2} Mg \cot \theta \quad (3)$$

(b) If the rod is uniform, its weight acts at its centre, so Mg acts at distance a from A and a from B.

$M(A)$: clockwise = anticlockwise
 $M_g \cos \theta(a) = S \sin \theta (2a)$
 $\frac{M_g \cos \theta}{2 \sin \theta} = S$
 $S = \frac{1}{2} M_g \cot \theta$

Given that $\tan \theta = \frac{3}{4}$

- (c) find the value of μ

(5)

(c) $\tan \theta = \frac{3}{4} \Rightarrow \sin \theta = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$
 $\Rightarrow \cos \theta = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$

$$R(\updownarrow): R_A = M_g \quad R(\leftrightarrow): S = F_r = \frac{1}{2} M_g \cot \theta$$

$$= \frac{1}{2} M_g \left(\frac{4}{3} \right)$$

$$= \frac{2}{3} M_g$$

In limiting equilibrium, $F_r = \mu R_A$

$$\frac{2}{3}Mg' = \mu Mg \Rightarrow \mu = \frac{2}{3}$$

- (d) find, in terms of M and g , the magnitude of the resultant force acting on the rod at A . (3)

(d)  $|resultant| = \sqrt{(Mg)^2 + (\frac{2}{3}Mg)^2}$
 $= \sqrt{\frac{13}{9}M^2g^2} = \frac{\sqrt{13}}{3}Mg$

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A .

A new value for S is calculated using this new model, with $\tan \theta = \frac{3}{4}$

- (e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

(1)

(e) The Moment of the rod's weight would be greater, because it would act at a greater distance from A.
The Moment of the Reaction at the wall would also need to be greater to balance the rod's Moment about A, so S would be larger.