



The diagram shows a model for the roof of a toy building. The roof is in the form of a solid triangular prism  $ABCDEF$ . The base  $ACFD$  of the roof is a horizontal rectangle, and the cross-section  $ABC$  of the roof is an isosceles triangle with  $AB = BC$ .

The lengths of  $AC$  and  $CF$  are  $2x$  cm and  $y$  cm respectively, and the height of  $BE$  above the base of the roof is  $x$  cm.

The total surface area of the **five** faces of the roof is  $600 \text{ cm}^2$  and the volume of the roof is  $V \text{ cm}^3$ .

- (a) Show that  $V = kx(300 - x^2)$ , where  $k = \sqrt{a} + b$  and  $a$  and  $b$  are integers to be determined. [6]
- (b) Use differentiation to determine the value of  $x$  for which the volume of the roof is a maximum. [4]
- (c) Find the maximum volume of the roof. Give your answer in  $\text{cm}^3$ , correct to the nearest integer. [1]
- (d) Explain why, for this roof,  $x$  must be less than a certain value, which you should state. [2]