



The diagram shows a sector  $OAB$  of a circle with centre  $O$  and radius  $OA$ . The angle  $AOB$  is  $\theta$  radians.  $M$  is the mid-point of  $OA$ . The ratio of areas  $OMB : MAB$  is  $2:3$ .

(a) Show that  $\theta = 1.25 \sin \theta$ .

[4]

The equation  $\theta = 1.25 \sin \theta$  has only one root for  $\theta > 0$ .

(b) This root can be found by using the iterative formula  $\theta_{n+1} = 1.25 \sin \theta_n$  with a starting value of  $\theta_1 = 0.5$ .

- Write down the values of  $\theta_2$ ,  $\theta_3$  and  $\theta_4$ .
- Hence find the value of this root correct to **3** significant figures.

[3]

(c) The diagram in the Printed Answer Booklet shows the graph of  $y = 1.25 \sin \theta$ , for  $0 \leq \theta \leq \pi$ .

- Use this diagram to show how the iterative process used in (b) converges to this root.
- State the type of convergence.

[3]

(d) Draw a suitable diagram to show why using an iterative process with the formula  $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$  does not converge to the root found in (b).

[2]

10(c)

