10

The diagram shows a sector OAB of a circle with centre O and radius OA. The angle AOB is θ radians. M is the mid-point of OA. The ratio of areas $OMB : MAB$ is $2:3$.

(a) Show that $\theta = 1.25 \sin \theta$. [4]

The equation $\theta = 1.25 \sin \theta$ has only one root for $\theta > 0$.

(b) This root can be found by using the iterative formula $\theta_{n+1} = 1.25 \sin \theta_n$ with a starting value of $\theta_1 = 0.5$.

- Write down the values of θ_2, θ_3 and θ_4.
- Hence find the value of this root correct to **3** significant figures. [3]

(c) The diagram in the Printed Answer Booklet shows the graph of $y = 1.25 \sin \theta$, for $0 \leqslant \theta \leqslant \pi$.

- Use this diagram to show how the iterative process used in **(b)** converges to this root.
- State the type of convergence. [3]

(d) Draw a suitable diagram to show why using an iterative process with the formula $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$ does not converge to the root found in **(b)**. [2]

10(c)