

5 Charlie claims to have proved the following statement.

“The sum of a square number and a prime number cannot be a square number.”

(i) Give an example to show that Charlie’s statement is not true. [1]

Charlie’s attempt at a proof is below.

Assume that the statement is not true.

\Rightarrow There exist integers n and m and a prime p such that $n^2 + p = m^2$.

$\Rightarrow p = m^2 - n^2$

$\Rightarrow p = (m - n)(m + n)$

$\Rightarrow p$ is the product of two integers.

$\Rightarrow p$ is not prime, which is a contradiction.

\Rightarrow Charlie’s statement is true.

(ii) Explain the error that Charlie has made. [1]

(iii) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number. [4]