8			**Summary method:** Express V in terms of h	**B1**	**3.3**	Correct substitution
			Differentiate V with respect to h	**M1**	**3.4**	NOT if $h = 50$ or $r = 50\tan30$ used
			Attempt chain rule, Attempt separate variables	**M1** **M1**		Resulting equation must involve exactly 2 variables Their equation must involve exactly 2 variables
			Correct integrals Substitute correct limits Answer	**A1** **M1** **A1**		Ignore limits Integrals must be of correct forms (see examples below) <u>Note 1</u> Candidates who substitute numerical values for h or V or r may be able to score the 2nd and/or 3rd M1 marks, but probably nothing else. See the example of this below. <u>Note 2</u>. There is a special case for candidates who use $r = h\sin30$ (answer $\frac{625\pi}{4}$ or 491). These can score all 4 M-marks and the final A1 <u>Note 3.</u> The chain rule may be used to find $\frac{dV}{dt}$ or $\frac{dh}{dt}$ or $\frac{dV}{dh}$ or $\frac{dV}{dr}$ or other derivatives. Two of the example methods below illustrate use of $\frac{dV}{dt}$ and $\frac{dV}{dr}$, but use of other derivatives can also lead to correct methods.

Question			Answer	Mark	AO	Guidance
8	ctd		**Example method 1**			
			$V = \frac{\pi}{3}(h\tan 30^{\circ})^2 h$ or $V = \frac{\pi}{3}\left(\frac{h}{\sqrt{3}}\right)^2 h$ oe	**B1**	**3.3**	or $V = \frac{\pi}{9}h^3$ oe
			$\frac{dV}{dh} = \frac{\pi}{3}h^2$	**M1**	**3.4**	Attempt differentiate their V in terms of h only NOT if $h = 50$ or $r = 50\tan 30$ used.
			$\frac{dV}{dt} = \frac{\pi}{3}h^2\frac{dh}{dt}$ oe or $\frac{dh}{dt} = \frac{3}{\pi h^2}\frac{dV}{dt}$	**M1**	**2.1**	Attempt use chain rule for $\frac{dV}{dt}$ or $\frac{dh}{dt}$ in terms of t & h only
			("$\frac{\pi}{3}h^2\frac{dh}{dt}$" $= -2h$ oe or $\frac{dh}{dt} = \frac{-6}{\pi h}$)			(Set their $\frac{dV}{dt} = -2h$)
			$\pi\int_{50}^{0} h\,dh = -\int_{0}^{t} 6\,dt$ oe	**M1**	**1.1**	Attempt separate variables in their equation in terms of h and t only (not V or r). Integral signs not essential
			$\left[\frac{\pi h^2}{2}\right]_{50}^{0} = \left[-6t\right]_{0}^{t}$ oe	**A1**	**2.1**	Correct integrals, any limits or none
			$-\pi \times \frac{50^2}{2} = -6t$	**M1**	**1.1**	Substitute correct limits into integrals of forms ah^2 & bt OR substitute $t = 0$ & $h = 50$ to find c <u>and</u> substitute $h = 0$
			Time = $\frac{625\pi}{3}$ secs or 654 secs (3 sf) oe	**A1**	**3.4**	Allow without secs or 10.9 mins or 10 mins 54 secs or: SC. Use of $r = h\sin 30$ (answer $\frac{625\pi}{4}$ or 491) can score <u>all 4 M-marks and final A1</u>
				[7]		

8	**ctd**		**Example method 2**			
			$V = \frac{\pi}{3}r^2\frac{r}{\tan 30^{\circ}}$ or $V = \frac{\pi}{\sqrt{3}}r^3$ oe	**B1**		Subst $h = \frac{r}{\tan 30^{\circ}}$ into correct formula for V
			$\frac{\mathrm{d}V}{\mathrm{d}r} = \sqrt{3}\pi r^2$	**M1**		
			$\frac{\mathrm{d}V}{\mathrm{d}t} = \sqrt{3}\pi r^2\frac{\mathrm{d}r}{\mathrm{d}t}$ oe	**M1**		Attempt use chain rule to find $\frac{\mathrm{d}V}{\mathrm{d}t}$ or $\frac{\mathrm{d}r}{\mathrm{d}t}$ in terms of t and r
			("$\sqrt{3}\pi r^2\frac{\mathrm{d}r}{\mathrm{d}t}$" $= -2r\sqrt{3}$ oe)			(Set their $\frac{\mathrm{d}V}{\mathrm{d}t} = -2r\sqrt{3}$ oe)
			$\pi\int_{\frac{50}{\sqrt{3}}}^{0} r\mathrm{d}r = -\int_{0}^{t} 2\mathrm{d}t$ oe	**M1**		Attempt separate variables in their equation in terms of r and t only (not V or h). Integral signs not essential
			$\left[\frac{\pi r^2}{2}\right]_{\frac{50}{\sqrt{3}}}^{0} = [-2t]_0^t$ oe	**A1**		Correct integrals, any limits or none
			$-\frac{\pi\times 50^2}{6} = -2t$	**M1**		Substitute correct limits into integrals of the form ar^2 & bt OR substitute $t = 0$ & $r = \frac{50}{\sqrt{3}}$ to find c <u>and</u> substitute $r = 0$
			Time $= \frac{625\pi}{3}$ secs or 654 secs (3 sf) oe	**A1**	**3.4**	Allow without secs or 10.9 mins or 10 mins 54 secs SC. Use of $r = h\sin 30$ (answer 491) can score M4A1

8	ctd		**Example method 3 (NOT using chain rule)**			**This method is different from the summary method above**
			$V = \frac{\pi}{3}(h\tan 30^{\circ})^2 h$ or $V = \frac{\pi}{3}\left(\frac{h}{\sqrt{3}}\right)^2 h$ oe	**B1**	3.3	or $V = \frac{\pi}{9}h^3$ oe
			$h = \sqrt[3]{\frac{9V}{\pi}}$	**M1**		Allow $h = kV^{1/3}$
			$\frac{dV}{dt} = -2 \times \sqrt[3]{\frac{9V}{\pi}}$	**M1**		$\frac{dV}{dt} = -2\times$(their h in terms of V)
			$\sqrt[3]{\frac{\pi}{9}} \int_{\frac{\pi 50^3}{9}}^{0} V^{-1/3}dV = -2[t]_0^t$	**M1**		Attempt separate variables in their equation in terms of V and t only (not h or r). Integral signs not essential
			$\sqrt[3]{\frac{\pi}{9}} \times \frac{3}{2}\left[V^{2/3}\right]_{\frac{\pi 50^3}{9}}^{0} = -2t$	**A1**		Correct integrals, any limits or none
			$-\sqrt[3]{\frac{\pi}{9}} \times \frac{3}{2} \times \left(\frac{\pi 50^3}{9}\right)^{2/3} = -2t$	**M1**		Substitute correct limits into integrals of forms $aV^{2/3}$ & bt OR substitute t=0 & $V = \frac{\pi 50^3}{9}$ to find c <u>and</u> substitute $V = 0$
			Time = $\frac{625\pi}{3}$ secs or 654 secs (3 sf) oe	**A1**		Allow without secs or 10.9 mins or 10 mins 54 secs or: SC. Use of $r = h\sin 30$ (answer $\frac{625\pi}{4}$ or 491) can score <u>all 4 M-marks and final A1</u>

8	ctd		**Example incorrect method** $r = 50/\sqrt{3}$ $V = \frac{\pi}{3} \times 2500 \times \frac{h}{3}$ $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{2500\pi}{9}$ $\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}h}$ $\frac{2500\pi}{9} = -2h\frac{\mathrm{d}t}{\mathrm{d}h}$ $\frac{\mathrm{d}h}{h} = -\frac{18}{2500\pi}\mathrm{d}t$	**B0** **M0** **M1** **M1**		