| Question | | | Answer | Mark | AO | Guidance |
|---|---|---|---|---|---|---|
| 7 | (a) | | $a^2 = 4b + 2$ | M1 | 2.1 | Setting up so that the deduction $a^2$ is even can be made. |
| | | | Hence $a^2$ is even. Hence $a$ is even | A1 | 2.2a | www, must see both statements and a convincing, correct, argument oe (e.g. $a^2 = 2(2b + 1)$ ) |
| | | | **Alternative method**<br>Assume that $a$ is odd, then $a^2$ is odd | M1 | | For setting up and stating that $a$ is odd $\Rightarrow a^2$ is odd<br>May see (not required) $a = 2n+1$, $a^2 = 2(2n^2+2n)+1$ Hence $a^2$ is odd |
| | | | $4b$ is even, so $a^2 - 4b$ is odd<br>Hence 2 is odd (so contradiction)<br>Hence $a$ is even. | A1 | | www, Must see both statements and a convincing, correct, argument |
| | | | | **[2]** | | |
| 7 | (b) | | Assume that $a^2 - 4b = 2$<br>Let $a = 2n$, (where $n$ is an integer) | M1 | 2.1 | Setting up (must see assumption and use of $a$ is even) |
| | | | Either of:<br>$4n^2 - 4b = 2$ / $4n^2 - 4b = 2$<br>$2n^2 - 2b = 1$, / $n^2 - b = 0.5$,<br>Hence 1 is even / $n^2 - b$ is an integer | A1 | 2.1 | Substituting in $a = 2n$ and correctly reaching an equation which shows a contradiction.<br>Accept the equivalent in words if clear and correct.<br>Also accept: $4n^2 - 4b$ is a multiple of 4, Hence $a^2 - 4b$ is a multiple of 4, which is a contradiction |
| | | | (Contradiction)<br>Hence $a^2 - 4b \neq 2$ | A1 | 2.2a | www, Must see both statements and a convincing, correct, argument |

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| | | | **Alternative method (1)**<br>Assume that $a^2 - 4b = 2$<br>$\Rightarrow a^2 = 4b + 2 = 2(2b + 1)$<br>For $a$ to be an integer, $2b + 1$ must be even<br>But ($2b$ is even, so) $2b + 1$ is odd<br>(Which is a contradiction) hence $a^2 - 4b \neq 2$ | <br><br>**M1**<br><br>**A1**<br>**A1** | | <br><br><br><br><br>Must see both statements and a convincing, correct, argument www |
| | | | **Alternative method (2)**<br>$a$ even $\Rightarrow a^2 = 4n$ ($n$ an integer)<br>$\Rightarrow a^2$ is congruent to 0 mod 4<br>$4b + 2$ is congruent to 2 mod 4<br>Therefore $a^2$ cannot equal $4b + 2$ | <br>**M1**<br><br>**A1**<br>**A1** | | <br><br><br><br>Must see previous two lines and a convincing, correct, argument www |
| | | | **Alternative method (3)**<br>Assume that $a^2 - 4b = 2$, then $a$ is even<br>(and consider whether $b$ is odd or even) | <br>**M1** | | <br>For setting up using part **(a)** and considering either case where $b$ is odd or even (ignore any reference to the cases where $a$ is odd as these are not required) May see (but not required) $a=2n$ so $a^2=4n^2$<br>Condone using the same letter (e.g. $n$) in $a$ and $b$ for this mark only. |
| | | | If $b$ is odd then 2 is either 0 or a multiple of 4, (so contradiction)<br>AND<br>If $b$ is even then 2 is either 0 or a multiple of 4, (so contradiction) | **A1** | | For correctly considering both cases either algebraically or in words.<br>May see (but not required) $b = 2m+1$, so $a^2 - 4b = 4(n^2 - (2m+1))$<br>And $b = 2m$, so $a^2 - 4b = 4(n^2 - 2m)$<br>Do not award this mark if same integer (e.g. $n$) used in both $a$ and $b$ |
| | | | Therefore $a^2$ cannot equal $4b + 2$ | **A1** | | A fully correct, convincing argument with conclusion, www. |
| | | | | **[3]** | | |