

**6 In this question you must show detailed reasoning.**

**(a) (i)** Use the formula for  $\cos(A+B)$ , and the double angle formulae, to show that  $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$ . **[2]**

**(ii)** Use this result to solve the equation  $4 \cos^3 \theta - 3 \cos \theta - \frac{\sqrt{2}}{2} = 0$  for  $0^\circ \leq \theta \leq 180^\circ$ . **[3]**

**(b) (i)** Show that  $\left(x + \frac{\sqrt{2}}{2}\right)(4x^2 - 2\sqrt{2}x - 1) = 4x^3 - 3x - \frac{\sqrt{2}}{2}$ . **[1]**

**(ii)** Hence find the exact roots of the equation  $4x^3 - 3x - \frac{\sqrt{2}}{2} = 0$ . **[2]**

**(c)** Use the results from parts **(a)(ii)** and **(b)(ii)** to show that  $\cos 15^\circ = \frac{\sqrt{2} + \sqrt{6}}{4}$ . **[2]**