

- 12 The discrete random variable  $X$  takes values 1, 2, 3, 4 and 5, and its probability distribution is defined as follows.

$$P(X=x) = \begin{cases} a & x = 1, \\ \frac{1}{2}P(X=x-1) & x = 2, 3, 4, 5, \\ 0 & \text{otherwise,} \end{cases}$$

where  $a$  is a constant.

- (i) Show that  $a = \frac{16}{31}$ . [2]

The discrete probability distribution for  $X$  is given in the table.

$x$	1	2	3	4	5
$P(X=x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

- (ii) Find the probability that  $X$  is odd. [1]

Two independent values of  $X$  are chosen, and their sum  $S$  is found.

- (iii) Find the probability that  $S$  is odd. [2]

- (iv) Find the probability that  $S$  is greater than 8, given that  $S$  is odd. [3]

Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable  $Y$  defined as follows.

$$P(Y=y+1) = \frac{1}{2}P(Y=y) \quad \text{for all positive integers } y.$$

- (v) Find  $P(Y=1)$ . [2]
- (vi) Give a reason why one of the variables,  $X$  or  $Y$ , might be more appropriate as a model for the number of attempts that Sheila needs to start her car. [1]