

13	(a)	$x = Ut$ $y = Vt - \frac{1}{2}gt^2$ $y = V\left(\frac{x}{U}\right) - \frac{1}{2}g\left(\frac{x}{U}\right)^2$ $y = \frac{Vx}{U} - \frac{gx^2}{2U^2} \Rightarrow 2U^2y = 2UVx - gx^2$	<b>M1*</b>  <b>M1dep*</b>  <b>A1</b>  <b>[3]</b>	<b>3.3</b>  <b>3.4</b>  <b>2.2a</b>	Setting up expressions for $x$ and $y$ using $s = ut + \frac{1}{2}at^2$ with $a = 0$ in $x$ and $\pm g$ in $y$ oe. <b>M0</b> if using $U$ instead of $V$ vertically  Eliminating both $t$ terms in $y$ to get an equation in $y, x, U, V$ (and possibly $g$ )  <b>AG</b> so sufficient working must be shown	Allow sign errors. May use $t = x / U$ in $y = Vt + \frac{1}{2}at^2$          www
13	(b)	$B \text{ passes through } \left(a, \frac{1}{2}a\right) \Rightarrow 2U^2\left(\frac{a}{2}\right) = 2UVa - ga^2$ $\left(\Rightarrow U^2 = 2UV - ga\right)$ $B \text{ passes through } (4a, 0) \Rightarrow 2UV(4a) - g(4a)^2 = 0$ $\left(\Rightarrow UV - 2ga = 0\right)$ $U = \sqrt{3ga}, V = \frac{2\sqrt{ga}}{\sqrt{3}} \text{ or } 2U = 3V$ $\tan \theta = \frac{V}{U} \Rightarrow \tan \theta = \frac{\frac{2}{\sqrt{3}}(\sqrt{ga})}{\sqrt{3ga}}$ $\tan \theta = \frac{2}{3} \Rightarrow \theta = 33.7^\circ (3 \text{ sf})$	<b>B1</b>  <b>B1</b>  <b>M1*</b>  <b>M1dep*</b>  <b>A1</b>  <b>[5]</b>	<b>3.4</b>  <b>3.1b</b>  <b>2.1</b>  <b>3.1b</b>  <b>2.2a</b>	Substituting $\left(a, \frac{1}{2}a\right)$ into given result from (a)  Substituting $(4a, 0)$ into given result from (a) – note that using $(3a, 0)$ is not a <b>MR</b>  Solve simultaneously (oe) to find either $U$ or $V$ (or their squares) in terms of $a$ and $g$ only, or for a linear equation (oe) in $V$ and $U$ only, if correct $2U = 3V$  Using $\tan \theta = \frac{V}{U}$ with their $U$ and their $V$  awrt 33.7 (an answer of 36.9 from using $(3a, 0)$ scores (if from correct working) <b>B1 B0 M1 M1 A0</b> )	oe e.g., if correct $V^2 = \frac{4}{3}ga$ and $U^2 = 3ga$

13	(c)		$\sqrt{3ga + \frac{4}{3}ga} = 54.6$ $a = 70.2$	<b>M1</b>  <b>A1</b>  [2]	<b>3.4</b>  <b>1.1</b>	Using $\sqrt{U^2 + V^2} = 54.6$ to set up an equation in $a$ (and possibly $g$ )  www awrt 70.2	an answer of 97.344 (awrt 97.3) from using $(3a, 0)$ in (b) scores <b>M1 A0</b>
			<b>Alternative 1</b>				
			$54.6 \cos(33.7\dots) = \sqrt{3ga}$ <b>or</b> $54.6 \sin(33.7\dots) = \sqrt{\frac{4}{3}ga}$	<b>M1</b>		Setting either the horizontal component equal to their $U$ (from (b)) or the vertical component equal to their $V$ (from (b)). Allow sin/cos confusion	<b>M0</b> for an unsupported value of $\theta$ (if used)
			$a = 70.2$	<b>A1</b>		www awrt 70.2	
				[2]			
			<b>Alternative 2</b>				
			$a = \frac{UV}{2g} = \frac{54.6 \cos(33.7\dots) \cdot 54.6 \sin(33.7\dots)}{2g}$	<b>M1</b>		Using their expression for $a$ (possibly seen in (b)) in terms of $U$ and $V$ with 54.6 and their $\theta$ . <b>M0</b> for an unsupported value of $\theta$	Allow sin/cos confusion
			$a = 70.2$	<b>A1</b>		www awrt 70.2	
				[2]			

13	(d)	$0 = V^2 - 2gH \left( \Rightarrow H = \frac{V^2}{2g} \right)$ $H = \frac{1}{2g} \left( \frac{4ga}{3} \right) = \frac{2}{3}a$ $H = \frac{2}{3}(70.2) = 46.8 \text{ (m)}$	<b>M1*</b>  <b>M1dep*</b>   <b>A1</b>  <b>[3]</b>	<b>3.3</b>  <b>3.4</b>  <b>2.2a</b>	Setting up the model using $v^2 = u^2 + 2as$ with $v = 0$ and $a = -g$  Using their expression for $V$ from (b) to get an expression for $H$ in terms of $a$ (oe) e.g., $H = \frac{V^2}{2g}$ where $V = 54.6 \sin \theta$ (allow $\cos \theta$ ) with their value of $\theta$ awrt 46.8	$H$ is the maximum height of $B$  <b>M0</b> for an unsupported value of $\theta$  an answer of 54.756 (awrt 54.8) from using $(3a, 0)$ in (b) scores <b>M1M1A0</b>
		<b>Alternative 1</b>				
		$0 = V - gt \Rightarrow t = \frac{54.6 \sin(33.7...)}{g}$	<b>M1*</b>		Find $t$ at maximum height with $v = 0$ , $a = -g$ and $u =$ their $V^1$ 54.6 (allow sin/cos confusion). <b>M0</b> for an unsupported value of $\theta$	$t = 3.090472522...$
		$H = (54.6 \sin(33.7...))t - \frac{1}{2}gt^2$ $= (54.6 \sin(33.7...))(3.09...) - \frac{1}{2}g(3.09...)^2$	<b>M1dep*</b>		Substituting their $t$ into $s = Vt - \frac{1}{2}gt^2$	$V^1$ 54.6 (allow sin/cos confusion)
		$H = 46.8 \text{ (m)}$	<b>A1</b>		awrt 46.8	
			<b>[3]</b>			
		<b>Alternative 2</b>				
		$x = 2a \Rightarrow 2U^2y = 4UVa - 4ga^2$	<b>M1*</b>		Setting $x = 2a$ or $1.5a$ and substituting into path equation from (a)	
		$2(54.6 \cos(33.7))^2 H =$ $4(54.6 \cos(33.7))(54.6 \sin(33.7))(70.2) - 4g(70.2)^2$	<b>M1dep*</b>		Substituting their $U$ , $V$ and $a$ to form an equation in $H$ (and possibly $g$ ) only. <b>M0</b> for an unsupported value of $\theta$	$U$ and $V^1$ 54.6 (allow sin/cos confusion)
		$H = 46.8 \text{ (m)}$	<b>A1</b>		awrt 46.8	
			<b>[3]</b>			

13	(e)	<p>examples of possible refinements include</p> <ul style="list-style-type: none"> <li>• taking into account the size of <math>B</math></li> <li>• taking into account that <math>B</math> is not a particle</li> <li>• taking into account the spin of <math>B</math></li> <li>• taking into account the dimensions of <math>B</math></li> <li>• taking into account the wind/weather</li> <li>• taking into account the thickness of the wall (which is assumed to be ‘thin’)</li> <li>• taking into account the clearance of the ball at the top of the wall</li> </ul>	B1	3.5c	<p>Allow any correct refinement, including use of a more accurate value of <math>g</math> rather than the assumed 9.8</p> <p><b>B0</b> if referring to</p> <ul style="list-style-type: none"> <li>• the mass or weight or shape of <math>B</math></li> <li>• the ground is unlikely to be horizontal</li> <li>• modelling the problem as three dimensional rather than two dimensional (unless specific detail given)</li> <li>• air resistance (only)</li> </ul> <p>If multiple refinements given, then all must be valid to score <b>B1</b></p>	
			[1]			