

<b>4</b>	<b>(a)</b>	$\frac{dy}{dx} = 3\sin 2x + 6x\cos 2x$ $\sin 2x + 2x\cos 2x = 0$ $\left(\frac{\sin 2x}{\cos 2x}\right) + 2x\left(\frac{\cos 2x}{\cos 2x}\right) = 0 \Rightarrow \tan 2x + 2x = 0$	<b>M1*</b> <b>M1dep*</b> <b>A1</b>	<b>2.1</b> <b>1.1</b> <b>2.2a</b>	Attempt use of product rule – answer of the form $\lambda\sin 2x + \mu x\cos 2x$ Sets derivative equal to zero <b>AG</b> – at least one step of correct intermediate working (e.g. $\cos 2x \tan 2x + 2x\cos 2x = 0$ ) from previous M mark to given answer (If $\cos 2x = 0$ and $\tan 2x + 2x = 0$ seen from $\cos 2x(\tan 2x + 2x) = 0$ then $\cos 2x = 0$ must be rejected)	$\lambda, \mu \neq 0$  Must be convincing as AG (must be = 0) – must see division by $\cos 2x$ (or stating the need to divide by this term but not just divide by $\cos$ )																																							
<b>4</b>	<b>(b)</b>	$f'(x) = 2\sec^2 2x + 2$ $x_{n+1} = x_n - \frac{\tan 2x_n + 2x_n}{2\sec^2 2x_n + 2}$ <table border="1" style="width: 100%; text-align: center;"> <thead> <tr> <th><math>x_0</math></th> <th><math>x_1</math></th> <th><math>x_2</math></th> </tr> </thead> <tbody> <tr><td>0.8</td><td>0.81389959...</td><td>0.839614100...</td></tr> <tr><td>0.9</td><td>0.96102142...</td><td>1.00372767...</td></tr> <tr><td>1.0</td><td>1.01365728...</td><td>1.01437714...</td></tr> <tr><td><math>\pi / 3</math></td><td>1.01096312...</td><td>1.01433905...</td></tr> <tr><td>1.1</td><td>0.99373628...</td><td>1.01287398...</td></tr> <tr><td>1.2</td><td>0.93865044...</td><td>0.99215748...</td></tr> <tr><td>1.3</td><td>0.87695335...</td><td>0.93545928...</td></tr> <tr><td>1.4</td><td>0.82520962...</td><td>0.85941601...</td></tr> <tr><td>1.5</td><td>0.79282139...</td><td>0.80006720...</td></tr> <tr><td>1.6</td><td>0.78677182...</td><td>0.78813953...</td></tr> <tr><td>1.7</td><td>0.81484089...</td><td>0.84130177...</td></tr> <tr><td>1.8</td><td>0.88770903...</td><td>0.94804414...</td></tr> </tbody> </table> <p>x-coordinate of P is 1.0144</p>	$x_0$	$x_1$	$x_2$	0.8	0.81389959...	0.839614100...	0.9	0.96102142...	1.00372767...	1.0	1.01365728...	1.01437714...	$\pi / 3$	1.01096312...	1.01433905...	1.1	0.99373628...	1.01287398...	1.2	0.93865044...	0.99215748...	1.3	0.87695335...	0.93545928...	1.4	0.82520962...	0.85941601...	1.5	0.79282139...	0.80006720...	1.6	0.78677182...	0.78813953...	1.7	0.81484089...	0.84130177...	1.8	0.88770903...	0.94804414...	<b>B1</b>  <b>M1</b>  <b>A1</b>	<b>1.1</b> <b>2.1</b>  <b>1.1</b> <b>2.2a</b>	Correct derivative Substitute their derivative (of the form $\alpha \sec^2 2x + 2$ ) into correct N-R formula  First two values correct ( $x_1$ and $x_2$ ) stated to at least 4 decimal places (truncated or rounded) – the table is not exhaustive and any other values used as a starting value in the interval given in the second guidance column will need to be checked - If no evidence of using NR (e.g. correct answer with no working) then no marks in this part. Dependent on correct NR formula (so must have scored B1)  Independent of previous A mark (but must have scored B1M1) – must be stated to exactly 4 decimal places	Need not be simplified and allow in terms of x only  Starting value must be in the interval $\frac{\pi}{4} < x_0 < \frac{3\pi}{4}$ ( $0.786 \leq x_0 \leq 2.35$ )  1.014378911 ...
$x_0$	$x_1$	$x_2$																																											
0.8	0.81389959...	0.839614100...																																											
0.9	0.96102142...	1.00372767...																																											
1.0	1.01365728...	1.01437714...																																											
$\pi / 3$	1.01096312...	1.01433905...																																											
1.1	0.99373628...	1.01287398...																																											
1.2	0.93865044...	0.99215748...																																											
1.3	0.87695335...	0.93545928...																																											
1.4	0.82520962...	0.85941601...																																											
1.5	0.79282139...	0.80006720...																																											
1.6	0.78677182...	0.78813953...																																											
1.7	0.81484089...	0.84130177...																																											
1.8	0.88770903...	0.94804414...																																											
			<b>[3]</b>	<b>[4]</b>																																									

		<b>ALT</b>	$f'(x) = 2 \cos 2x + 2 \cos 2x - 4x \sin 2x$ $f'(x) = 6 \cos 2x + 6 \cos 2x - 12x \sin 2x$	B1		Correct derivative of either $\sin 2x + 2x \cos 2x$ or $3 \sin 2x + 6x \cos 2x$																																											
			$x_{n+1} = x_n - \frac{\sin 2x_n + 2x_n \cos 2x_n}{4 \cos 2x_n - 4x_n \sin 2x_n}$	M1		Substitute their derivative (of the form $\alpha \cos 2x + \beta x \sin 2x$ ) into correct formula for N-R	Need not be simplified and allow in terms of $x$ only																																										
			<table border="1"> <thead> <tr> <th><math>x_0</math></th> <th><math>x_1</math></th> <th><math>x_2</math></th> </tr> </thead> <tbody> <tr><td>0.65</td><td>1.56363958...</td><td>0.80243617...</td></tr> <tr><td>0.7</td><td>1.28834722...</td><td>1.02095799...</td></tr> <tr><td>0.75</td><td>1.15730247...</td><td>1.02135573...</td></tr> <tr><td>0.8</td><td>1.08739962...</td><td>1.01697831...</td></tr> <tr><td>0.9</td><td>1.02795637...</td><td>1.01449472...</td></tr> <tr><td>1.0</td><td>1.01452414...</td><td>1.01437893...</td></tr> <tr><td><math>\pi/3</math></td><td>1.01500404...</td><td>1.01437917...</td></tr> <tr><td>1.1</td><td>1.01775095...</td><td>1.01438636...</td></tr> <tr><td>1.2</td><td>1.02326915...</td><td>1.01442951...</td></tr> <tr><td>1.3</td><td>1.01965227...</td><td>1.01439698...</td></tr> <tr><td>1.4</td><td>0.99197402...</td><td>1.01474280...</td></tr> <tr><td>1.5</td><td>0.91147495...</td><td>1.02488159...</td></tr> <tr><td>1.6</td><td>0.70130089...</td><td>1.28368799...</td></tr> </tbody> </table>	$x_0$	$x_1$	$x_2$	0.65	1.56363958...	0.80243617...	0.7	1.28834722...	1.02095799...	0.75	1.15730247...	1.02135573...	0.8	1.08739962...	1.01697831...	0.9	1.02795637...	1.01449472...	1.0	1.01452414...	1.01437893...	$\pi/3$	1.01500404...	1.01437917...	1.1	1.01775095...	1.01438636...	1.2	1.02326915...	1.01442951...	1.3	1.01965227...	1.01439698...	1.4	0.99197402...	1.01474280...	1.5	0.91147495...	1.02488159...	1.6	0.70130089...	1.28368799...	A1		First two values correct ( $x_1$ and $x_2$ ) stated to at least 4 decimal places – the table is not exhaustive and any other values used as a starting value in the interval given in the second guidance column will need to be checked - If no evidence of using NR (e.g. correct answer with no working) then no marks in this part. Dependent on correct NR formula (so must have scored B1)	Starting values must be in interval $0.65 \leq x_0 \leq 1.61$
$x_0$	$x_1$	$x_2$																																															
0.65	1.56363958...	0.80243617...																																															
0.7	1.28834722...	1.02095799...																																															
0.75	1.15730247...	1.02135573...																																															
0.8	1.08739962...	1.01697831...																																															
0.9	1.02795637...	1.01449472...																																															
1.0	1.01452414...	1.01437893...																																															
$\pi/3$	1.01500404...	1.01437917...																																															
1.1	1.01775095...	1.01438636...																																															
1.2	1.02326915...	1.01442951...																																															
1.3	1.01965227...	1.01439698...																																															
1.4	0.99197402...	1.01474280...																																															
1.5	0.91147495...	1.02488159...																																															
1.6	0.70130089...	1.28368799...																																															
			$x$ -coordinate of $P$ is 1.0144	A1 [4]		Independent of previous A mark (but must have scored B1M1) – must be stated to exactly 4 decimal places	1.014378911 ...																																										

4	(c)		$h = \frac{\pi}{8}$ $\frac{1}{2}h \left[ 0 + 2 \left( 3 \left( \frac{\pi}{8} \right) \sin \left( \frac{\pi}{4} \right) + 3 \left( \frac{\pi}{4} \right) \sin \left( \frac{\pi}{2} \right) + 3 \left( \frac{3\pi}{8} \right) \sin \left( \frac{3\pi}{4} \right) \right) + 0 \right]$ $\left( = \frac{1}{2}h \left[ 0 + 2 \left( \frac{3}{16} \pi \sqrt{2} + \frac{3}{4} \pi + \frac{9}{16} \pi \sqrt{2} \right) + 0 \right] \right)$ $\frac{1}{16} \pi \left( \frac{3}{8} \pi \sqrt{2} + \frac{3}{2} \pi + \frac{9}{8} \pi \sqrt{2} \right)$ $\frac{3}{32} \pi^2 (\sqrt{2} + 1)$	<b>B1</b>  <b>M1</b>    <b>A1</b>  <b>A1</b> <b>[4]</b>	<b>1.1</b> For using $\frac{1}{2} \times \frac{\pi}{8}$ or $\frac{\pi}{16}$ or exact equivalent or for stating $h$  <b>2.1</b> Correct [...] structure including multiplying the middle terms by 2. The zeros may be omitted. Allow one incorrect $y$ value only. Any additional values or repeated values is M0. M0 if using $x$ values or if only non-exact values seen but allow for the M mark if left in terms of sin    <b>1.1</b> Correct (possibly un-simplified) exact expression for integral  <b>2.2a</b> $k = \frac{3}{32}$ www	Not just for $\frac{\pi}{8}$ seen  Ignore $\frac{1}{2}h$ term for this mark  Note first 0 might be $3(0)\sin(2(0))$ and second 0 might be $3\left(\frac{\pi}{2}\right)\sin\pi$  Not in terms of sin and correct value of $h$ used
4	(d)	(i)	$\int_0^{\frac{1}{2}\pi} 3x \sin 2x \, dx = \frac{3}{4} \pi$	<b>B1</b>  <b>[1]</b>	<b>1.1</b> <b>BC</b> – ignore any working and mark final answer only (allow awrt 2.36)	oe, e.g. 2.356...
4	(d)	(ii)	$\frac{3}{32} \pi^2 (\sqrt{2} + 1) \approx 2.23 < 2.356\dots$ so trapezium rule gives an under-estimate of the area	<b>B1</b>  <b>[1]</b>	<b>2.2a</b> Dependent on correct value in (c) (but may not be exact) and correct value for integral in (d)(i) – must state in this part correct decimal values (to at least 2 sf) for comparison (or 2.36 seen in (d)(i))	B0 if only ‘under-estimate’ stated with no reasoning
4	(d)	(iii)	LH trapezium above curve, but others below curve, so overall approximation not clear	<b>B1</b>	<b>2.4</b> oe e.g. trapezia/strips not all below the curve e.g. the curve changes from being convex to concave (concave up to concave down) e.g. the rate of change of the gradient changes from positive to negative	Condone mention of the curve being both concave and convex in the interval

