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The diagram shows the curve C with parametric equations

$x = \dfrac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. [4]

The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

(b) Show that the area of R is given by

$$\int_a^b 3t e^{-2t}\, dt,$$

where a and b are constants to be determined. [3]

(c) Hence determine the exact area of R. [5]