Question			Answer	Marks	AO	Guidance	
5	(a)		**DR**				
			$y=(2x-3)\left(4x^2+1\right)^{-1}$ $\Rightarrow \dfrac{dy}{dx}=2\left(4x^2+1\right)^{-1}+(2x-3)(-1)\left(4x^2+1\right)^{-2}(8x)$	**M1***	**1.1**	Attempt use of quotient rule or equivalent (e.g. product rule). Condone one incorrect term only (of the five terms) but must be subtraction in the numerator (but allow subtraction the wrong way round); condone absence of brackets; no denominator (if using quotient rule) is **M0**	By the five terms we mean the four in the numerator and the fifth is the term in the denominator
			$y=\dfrac{2x-3}{4x^2+1}\Rightarrow\dfrac{dy}{dx}=\dfrac{\left(4x^2+1\right)(2)-(2x-3)(8x)}{\left(4x^2+1\right)^2}$	**A1**	**1.1**	cao must include brackets as necessary	Any correct equivalent form
			$\dfrac{2\left(1+12x-4x^2\right)}{\left(4x^2+1\right)^2}=2$	**M1dep***	**3.1a**	Sets their derivative (in any form) equal to 2 (**M0** if equating to normal gradient)	May equate at any stage (even after incorrect manipulation of their derivative)
			$1+12x-4x^2=\left(4x^2+1\right)^2\Rightarrow 16x^4+12x^2-12x=0$	**M1**	**1.1**	Multiply both sides by $\left(4x^2+1\right)^2$ and simplify (so combining like terms) to obtain a quartic equation (must be expanded with at least three terms – condone lack of = 0 if all terms on the same side) – allow sign errors/minor slips but the expansion of $\left(4x^2+1\right)^2$ must be three terms of the form $16x^4+ax^2+1$ where $a=\pm 4,\pm 8$	**Dependent on both previous M marks**
			$x\left(4x^3+3x-3\right)=0\Rightarrow 4x^3+3x-3=0$ as $x\neq 0$	**A1**	**2.3**	**AG** with explicit rejection of $x = 0$ – as a minimum must indicate that x cannot equal 0	Just cancelling x is **A0**
				[5]			

Question			Answer	Marks	AO	Guidance	
5	(b)		**DR** Consider both $f(0.5)$ and $f(1)$ Where $f(x) = \pm(4x^3 + 3x - 3)$	M1	1.1	Working or correct answer for one value is sufficient evidence of correct method but both 0.5 and 1 must be seen	Just stating that $f(0.5) < 0$ and $f(1) > 0$ is **M0**
			$f(0.5) = -1 < 0$ and $f(1) = 4 > 0$ (or $f(0.5) = 1 > 0$ and $f(1) = -4 < 0$) Change of sign indicates that the x-coordinate lies between 0.5 and 1	A1 **[2]**	2.4	Correct values together with explanation (change of sign) and correct conclusion (as a minimum 'root' oe)	
			Alternative				
			Considers both $g(0.5)$ and $g(1)$ where $g(x) = \dfrac{(4x^2+1)(2) - (2x-3)(8x)}{(4x^2+1)^2}$	M1		Must be using the correct derivative. Working or correct answer for one value is sufficient evidence of correct method but both 0.5 and 1 must be seen	Just stating that $g(0.5) > 2$ and $g(1) < 2$ is **M0**
			$g(0.5) = 3 > 2$ and $g(1) = 0.72 < 2$ Values either side of 2 indicates that the x-coordinate lies between 0.5 and 1	A1		Correct values together with explanation (values either side of 2) and correct conclusion (as a minimum 'root' oe)	
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(c)		**DR**				
			Let $\mathrm{h}(x)=\dfrac{3-4x^3}{3} \Rightarrow \mathrm{h}'(x)=-4x^2$	**B1***	**2.1**	Calculates correct derivative of rhs of given iterative formula	
			As the root α lies in the interval (0.5, 1) $\Rightarrow \mathrm{h}'(\alpha)<-1$ so iterative formula cannot converge to the *x*-coordinate of *P*	**B1dep***	**2.2a**	Correct explanation that any value in the given interval gives a gradient which is less than –1	**No marks** for just showing that the iteration doesn’t converge using different starting values
				[2]			
5	(d)		**DR**				
			$\mathrm{f}(x_n)=4x_n^3+3x_n-3 \Rightarrow \mathrm{f}'(x_n)=12x_n^2+3$	**B1**	**1.1**	Correct derivative (possibly seen in N-R formula)	Condone *x* for x_n oe
			$x_{n+1}=x_n-\left\{\dfrac{4x_n^3+3x_n-3}{12x_n^2+3}\right\}$	**M1**	**2.1**	Correct N-R formula seen with correct $\mathrm{f}(x_n)$ and their $\mathrm{f}'(x_n)$ substituted	Condone *x* for x_n oe
			$x_0=0.5$, $x_1=\frac{2}{3}$ or 0.666666..., $x_2=\frac{29}{45}$ or 0.644444..., ($x_3=0.64395510...$)	**A1**	**1.1**	First two iterations correctly stated to at least 5 decimal places (or exact) (truncated or rounded)	The correct first two iterations can imply **B1 M1**
			x coordinate of *P* is 0.64395	**A1**	**2.2a**	Independent of previous **A** mark (but must have scored **B1 M1**) – must be stated to exactly **5** decimal places	This **A** mark does not imply the previous **A** mark
			y coordinate of *P* is –0.64395	**B1**	**1.1**	Independent of all previous marks – must be stated to exactly **5** decimal places	The correct answers with no evidence of N-R (e.g. no iterations stated **and** no N-R formula) then **B0M0A0A0B1** max.
				[5]			