

7

(a)

DR

$$m \sec \theta + 3 \cos \theta = 4 \sin \theta$$

$$\left(\Rightarrow m \sec \theta + \frac{3}{\sec \theta} = 4 \sin \theta \right)$$

$$m \sec^2 \theta + 3 = 4 \sin \theta \sec \theta$$

$$m(1 + \tan^2 \theta) + 3 = 4 \tan \theta$$

$$m + m \tan^2 \theta + 3 = 4 \tan \theta$$

$$\Rightarrow m \tan^2 \theta - 4 \tan \theta + (m + 3) = 0$$

M1**2.1**Or for $m \sec \theta + 3 \cos \theta = 4 \sin \theta$

$$\left(\Rightarrow \frac{m \sec \theta}{\cos \theta} + 3 = 4 \frac{\sin \theta}{\cos \theta} \right)$$

$$m \sec^2 \theta + 3 = 4 \tan \theta$$

The first **M** mark is for a valid method arriving at a three term equation containing $\sec^2 \theta$

M1**1.1**

Correctly uses the identity

$1 + \tan^2 \theta \equiv \sec^2 \theta$ to obtain an equation in $\tan \theta$ only

A1**2.2a****AG** so sufficient working must be shown**[3]**

Squaring each individual term of the original equation scores no marks

A0 if angle missing from any trigonometric terms

7	(b)	<p>DR</p> $\Delta = (-4)^2 - 4m(m+3)$ <p>As the quadratic equation in \tan has only one solution for θ in the given interval (and as the range of \tan in the given interval is all non-zero real values) this implies that the given equation must only have one real root and therefore</p> $(-4)^2 - 4m(m+3) = 0 \Rightarrow m^2 + 3m - 4 = 0$ $(m+4)(m-1) = 0 \Rightarrow m = -4 \text{ only as } m \text{ is a negative integer}$ $m = -4 \Rightarrow (2 \tan \theta + 1)^2 = 0 \text{ so } \tan \theta = -0.5$ $\theta = 2.68 \text{ (3 sf)}$	<p>M1*</p> <p>M1dep*</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>3.1a</p> <p>1.1</p> <p>1.1</p> <p>1.1</p> <p>2.4</p>	<p>Considers discriminant of given quadratic equation in \tan (c must be two terms) to get an expression in m only. M0 for embedded discriminant in quadratic formula unless explicitly considered</p> <p>Sets their discriminant equal to zero and obtains an expanded three-term quadratic in m</p> <p>State or imply $m = -4$ only</p> <p>Uses their negative integer value of m, and solves the equivalent of their three term quadratic equation in \tan, to obtain (at least) $\tan \theta = k$ - dependent on both previous M marks Allow - $4 \tan^2 \theta - 4 \tan \theta - 1 = 0$ P $\tan \theta = -0.5$ for this mark</p> <p>For full marks must explain why the discriminant should be set equal to zero – must say that as there is only one value of θ or $\tan \theta$ P $\Delta = 0$ (as a minimum must see explicit mention of ‘one’ together with ‘θ’ or ‘$\tan \theta$’ for this mark). Allow awrt 2.68</p>	<p>Allow $4^2 - 4m(m+3)$</p> <p>Reasoning for setting the discriminant equal to zero is not required for this mark</p> <p>where k is non-zero. If no method shown for solving their quadratic, then award this mark if the solution is correct for their quadratic</p> <p>2.677945045...</p>
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