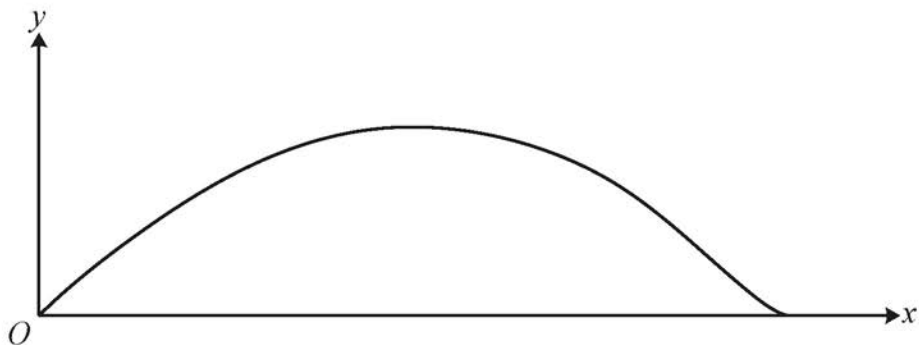


- 5 A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.



The emblem is modelled by the region between the  $x$ -axis and the curve with parametric equations

$$x = 1 + 0.2t - \cos t, \quad y = k \sin^2 t,$$

where  $k$  is a positive constant and  $0 \leq t \leq \pi$ .

Lengths are in metres and the area of the emblem must be  $1 \text{ m}^2$ .

(a) Show that  $k \int_0^\pi (0.2 + \sin t - 0.2 \cos^2 t - \sin t \cos^2 t) dt = 1$ . [3]

(b) Determine the exact value of  $k$ . [6]