Question			Answer	Marks	AO	Guidance	
4	(a)		$2\cot^2 x - 9\operatorname{cosec} x - 3[=0]$ $2\left(\frac{\cos^2 x}{\sin^2 x}\right) - 9\left(\frac{1}{\sin x}\right) - 3[=0]$	**M1**	**2.1**	Use of both $\cot x \equiv \frac{\cos x}{\sin x}$ and $\operatorname{cosec} x \equiv \frac{1}{\sin x}$	Condone use of s and c throughout but final answer must be in terms of sine – allow e.g. θ for x for **M** marks but must be x for the **A** mark
			$2\cos^2 x - 9\sin x - 3\sin^2 x[=0]$ $2(1-\sin^2 x) - 9\sin x - 3\sin^2 x[=0]$	**M1**	**1.1**	Correct use of $\sin^2 x + \cos^2 x \equiv 1$ to obtain an equation in $\sin x$ only	**Not dependent on the first M mark**
			$2 - 2\sin^2 x - 9\sin x - 3\sin^2 x[=0]$ $\Rightarrow 5\sin^2 x + 9\sin x - 2 = 0$	**A1** **[3]**	**2.2a**	**AG** (so must be equal to zero) – sufficient working must be shown – any errors seen is **A0**	**A0** if an angle is missing from any trig. expression **used in their working**
			Alternative method $2(\operatorname{cosec}^2 x - 1) - 9\operatorname{cosec} x - 3[=0]$	**M1**		Correct use of $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$	
			$2\operatorname{cosec}^2 x - 9\operatorname{cosec} x - 5[=0]$ $\Leftrightarrow \frac{2}{\sin^2 x} - \frac{9}{\sin x} - 5[=0]$	**M1**		Replacing $\operatorname{cosec} x$ with $\frac{1}{\sin x}$ to obtain an equation in $\sin x$ only Note: $2\operatorname{cosec}^2 x - 9\operatorname{cosec} x - 5 = 0$ $\Rightarrow 2 - 9\sin x - 5\sin^2 x = 0$ with no intermediate working is **M0** unless explicit mention is made of multiplying through by $\sin^2 x$	**Not dependent on first M mark** or explicit mention is made of dividing by $\operatorname{cosec}^2 x$
			$2 - 9\sin x - 5\sin^2 x[=0]$ $\Rightarrow 5\sin^2 x + 9\sin x - 2 = 0$	**A1**		**AG** (so must be equal to zero) – sufficient working must be shown – any errors seen is **A0**	**A0** if an angle is missing from any trig. expression used in their working

Question			Answer	Marks	AO	Guidance	
4	(b)	(i)	**DR**				
			$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3[=0]$ $\Rightarrow (5\sin 2\theta - 1)(\sin 2\theta + 2)[=0]$	**M1**	**1.1**	**SEE APPENDIX for awarding this mark (solving 3TQ expressions)**	condone using x for θ **or** 2θ for the **M** mark – condone for **M1** only $(5\sin\theta - 1)(\sin\theta + 2)$
			$\sin 2\theta = 0.2$ only as $\sin 2\theta \neq -2$	**B1**	**2.3**	Correctly stating that $\sin 2\theta = 0.2$ and that $\sin 2\theta$ cannot equal -2 (must explicitly reject the -2 (but no rationale required) - this mark is not implied by correct values for θ (as **DR** required)	Must be solving $5\sin^2 2\theta + 9\sin 2\theta - 2 = 0$ for the **B** marks condone $\sin 2x = 0.2$
			$[\theta =]\ 0.101$	**B1**	**1.1**	awrt 0.101 (0.1006789…) www	
			$[\theta =]\ 1.470$	**B1**	**1.1**	awrt 1.470 (1.4701173…) www	
						Ignore additional solutions outside of the range $0 < \theta < \pi$, but if any other solutions inside the range, award at most one of the two final **B** marks for one correct value	**SC B1** for awrt 0.10 **and** awrt 1.47 only if 3 dp or better) not seen **SC B1** for awrt 5.77 **and** awrt 84.2 only (working in degrees)
				[4]			

Question			Answer	Marks	AO	Guidance	
4	(b)	(ii)	$5\sin^2 2\theta + 9\sin 2\theta - 2[=0]$ $\Rightarrow 5(2\theta)^2 + 9(2\theta) - 2[=0]$ $\left(10\theta^2 + 9\theta - 1[=0]\right)$	M1	1.2	Use of the small angle approximation $\sin 2\theta \approx 2\theta$ twice in the given answer from **(a)** to obtain a three-term quadratic in θ (allow un-simplified)	Award **M1** only for $5\theta^2 + 9\theta - 2[=0]$ (so for using θ instead of 2θ) – allow e.g. x for θ
			$(10\theta - 1)(\theta + 1) = 0 \Rightarrow \theta = 0.10(000...)$ so is accurate to 2 decimal places	A1 **[2]**	2.4	State 0.10 (or better e.g. 0.100…) **as a decimal** following a correct quadratic in θ seen (no method required for solving the quadratic) and comment that this is accurate to 2 dp (**as a minimum** must mention '2 dp' with the value of 0.10(000…) appearing in this part and 0.10 **or** 0.101 **or** 0.100(6789…) appearing in part **(b)(i)**)	**This mark is dependent on an awrt 0.10 seen in part (b)(i) or a correct sign change test (see below)** **Ignore any consideration of other root(s)** **SEE APPENDIX FOR ALTERNATIVE**
			Alternative for M mark $\sin 2\theta = 0.2$ (from part **(b)(i)**) $\Rightarrow 2\theta = 0.2$			Re-writing at least one of their equations $\sin 2\theta = k$ with $-1 < k < 1$ (from part **(a)**) as $2\theta = k$	
			Alternative for A mark $f(\theta) = 2\cot^2 2\theta - 9\,\mathrm{cosec}\, 2\theta - 3$ $f(0.105) = -2.1497... < 0$ $f(0.095) = 3.4185... > 0$ Change of sign indicates that the approximate solution is accurate to 2 decimal places			Correct values to at least 1 dp (rot) with explanation ('change of sign' either stated **or** comparing values with zero) **and** correct conclusion (as a minimum must mention '2 dp')	

APPENDIX

Rules for solving quadratics in questions 2 and 4(b)(i) ONLY

In questions **2** and **4(b)(i)** candidates are required to solve 3 term quadratics (3TQ) using **DR** – therefore we must see a correct, complete method for solving these quadratics – the correct answers do not imply the corresponding M mark, for example in question 2, $9x^2-38x+8=0 \Rightarrow x=4$ or $x=\frac{2}{9}$ is **M0**

Rules for factorising:

$at^2+bt+c \Rightarrow (mt+n)(pt+q)$ where $a=mp$ **and** one of $mq+np=b$ **or** $c=nq$ (so when expanding their factorised expression it must give the correct quadratic term and one other term correct of the preceding 3TQ expression/equation)

e.g. in question **2** (and similarly for question **4(b)(i)**):

$9x^2-38x+8=(x-\frac{2}{9})(x-4)$ is **M0** (but the following **B1** for the correct c.v. of $\frac{2}{9}$ and 4 in qu. 2 can still be awarded as they follow from these two factors)

$9x^2-38x+8=(3x+8)(3x+1)$ is **M1** (when expanded the x^2 and constant terms are correct)

Allow correct part factorisation for their 3TQ expression e.g. if correct 3TQ then in question 2 the expression $9x(x-4)-2(x-4)$ scores **M1**

Rules for the formula:

Must apply the correct formula for their three-term quadratic (no errors even if correct formula is stated) – note that stating the formula (in terms of *a*, *b* and *c*) followed immediately by the corresponding roots is **M0** – we **must** see the formula being applied e.g. $9x^2-38x+8=0 \Rightarrow x=\dfrac{38\pm\sqrt{38^2-4(9)(8)}}{2(9)}$.

Minimal acceptable working would be $x=\dfrac{38\pm\sqrt{1156}}{18}$ (so must explicitly see the discriminant) for **M1**

Rules for completing the square – using $9x^2-38x+8=0$ as an example:

The **M1** is not awarded until correctly getting to the stage of $x-\dfrac{19}{9}=\pm\sqrt{\dfrac{289}{81}}$ (must include $\pm$ so implying two roots) with no errors (so consistent with applying the formula correctly)

Alternative for 4(b)(ii)		
$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3[=0] \Rightarrow \frac{2}{(2\theta)^2} - \frac{9}{2\theta} - 3\,[=0]$	**M1**	Use of small angle approximations $\sin 2\theta \approx 2\theta$ and $\tan 2\theta \approx 2\theta$ in the given equation from **(a)** to obtain what would be equivalent to a three-term quadratic in θ (allow un-simplified). Allow **M1** only for $\frac{2}{\theta^2} - \frac{9}{\theta} - 3\,[=0]$ (so for using θ instead of 2θ) – allow e.g. x for θ
$6\theta^2 + 9\theta - 1 = 0 \Rightarrow \theta = 0.10(39...)$ so is accurate to 2 dp	**A1**	State 0.10 (or better e.g. 0.1039…) as a decimal following a correct quadratic in θ (no method required for solving quadratic) and comment that this is accurate to 2 dp (as a minimum must mention '2 dp' with the value of 0.10(39…) appearing in this part and 0.10 or 0.101 or 0.100(6789…) appearing in part **(b)(i)**