

Question			Answer	Marks	AO	Guidance	
5	(a)	(i)	$y = (x^3 - 2x^2) \ln x \Rightarrow \frac{dy}{dx} = \dots$	M1	2.1	M1 for attempt to differentiate using the product rule (oe) – answer must be of the form $(x^3 - 2x^2) \times \frac{k_1}{x} + (k_2x^2 + k_3x) \ln x$ for non-zero constants k_1, k_2, k_3	Condone invisible brackets for this mark
			$\frac{dy}{dx} = (x^3 - 2x^2) \left(\frac{1}{x} \right) + (3x^2 - 4x) \ln x$	A1	1.1	A1 for a correct first derivative (allow un-simplified)	Condone invisible brackets but only if correctly recovered at some stage
			$\frac{d^2y}{dx^2} = 2x - 2 + \frac{3x^2 - 4x}{x} + (6x - 4) \ln x$	A1	1.1	A1 for a correct second derivative (allow un-simplified)	Condone invisible brackets but only if correctly recovered at some stage
			$\frac{d^2y}{dx^2} = 0 \Rightarrow 2x - 2 + \frac{3x^2 - 4x}{x} + (6x - 4) \ln x = 0$	M1	1.1	Setting the second derivative (which if simplified would be of the form $ax + b + (cx + d) \ln x$ with non-zero constants a, b, c and d) equal to zero	
			$5x - 6 = (4 - 6x) \ln x \Rightarrow x = \frac{6 + (4 - 6x) \ln x}{5}$	A1	2.2a	AG – so sufficient working must be shown – at least one intermediate line of working from second derivative set equal to zero to given answer	Any errors seen (e.g. any missing/invisible brackets) is A0
				[5]			

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5	(a)	(ii)	$x_{n+1} = \frac{6 + (4 - 6x_n) \ln x_n}{5}$ $x_1 = 1.1$ $x_2 = 1.150438...$ $x_3 = 1.118643...$ $x_4 = 1.139191...$ $x_5 = 1.126105...$ $x_6 = 1.134521...$	B1	1.1	Uses given result and given starting value (of 1.1) to obtain correct x_2 and x_3 (so first two iterations after the initial value of 1.1) to at least 2 dp (rot) – but all stated values in these two terms must be correct	
			x -coordinate of M is 1.13	B1	2.2a	Must be stated to 2 dp only – not dependent on the first B mark – can be awarded if either of x_2 and x_3 are incorrect (assume that the iterative process corrected itself or a slip in the candidate writing down an earlier value)	Must be clear that x is 1.13 (and not the final term shown in the iterative process e.g. $x_6 = 1.13$ only is B0) – this mark can be awarded from using alternative iterative methods e.g. Newton-Raphson
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(b)		Curve crosses the x-axis at 1 and 2 $\int (x^3 - 2x^2) \ln x \, dx = \dots$ $= \left(\frac{1}{4}x^4 - \frac{2}{3}x^3 \right) \ln x - \int \left(\frac{1}{4}x^4 - \frac{2}{3}x^3 \right) \left(\frac{1}{x} \right) dx$ $= \left(\frac{x^4}{4} - \frac{2x^3}{3} \right) \ln x - \frac{x^4}{16} + \frac{2x^3}{9} (+c)$ $\left\{ \left(\frac{16}{4} - \frac{16}{3} \right) \ln 2 - 1 + \frac{16}{9} \right\} - \left\{ 0 - \frac{1}{16} + \frac{2}{9} \right\}$ $\int_1^2 (x^3 - 2x^2) \ln x \, dx = -\frac{4}{3} \ln 2 + \frac{89}{144}$ $\Rightarrow \text{Area} = \frac{4}{3} \ln 2 - \frac{89}{144}$	B1* M1* A1 A1 M1dep* A1 [6]	3.1a 2.1 1.1 1.1 1.1 3.2a	Correct x-intercepts (soi) – ignore mention of $x = 0$ M1 for attempt at integration by parts – must be of the form $(ax^4 + bx^3) \ln x \pm \int (cx^4 + dx^3) \times \frac{1}{x} (dx)$ for non-zero constants a, b, c and d correct first application (allow un-simplified) cao (allow un-simplified) Uses correct limits completely correctly $\pm(F(2) - F(1))$ in their fully integrated expression – need not be simplified (or equivalent) Must be of this form but allow exact equivalents (for example, $\frac{1}{3} \ln 16 - \frac{89}{144}$) but the p and r must be positive rational numbers and q must be a positive integer Be aware of those who consider $\int_1^2 (2x^2 - x^3) \ln x \, dx$ which is correct	Could be seen as limits on integral(s) Limits not required for this and the next two A marks (so condone incorrect limits too for these 3 marks) dx not required and integral sign(s) can be implied Do not condone invisible brackets unless recovered For reference – one possibility is: $p = \frac{4}{3}, q = 2, r = \frac{89}{144}$ These values do not need to be stated explicitly