

| Question |  |  | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Marks                                                                                                                                         | AO                                                                                                            | Guidance                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
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| 7        |  |  | <p>For reference: <math>5x - 2xy + 2y^2 - k = 0</math></p> <p><math>5 - 2y - 2x \frac{dy}{dx} + 4y \frac{dy}{dx} = 0</math></p> <p><math>\frac{dy}{dx} = \frac{2y - 5}{4y - 2x}</math> so at <math>P</math> and <math>Q</math>, <math>4y - 2x = 0</math></p> <p><math>5x - 2xy + 2y^2 - k [= 0]</math><br/> <math>\Rightarrow 5(2y) - 2(2y)y + 2y^2 - k [= 0]</math><br/> <math>\Rightarrow 2y^2 - 10y + k [= 0]</math></p> <p>Difference in <math>y</math> values is 3 so <math>\frac{\sqrt{100 - 8k}}{2} = 3</math><br/> <math>\left( \text{or } \frac{10 + \sqrt{100 - 8k}}{4} - \frac{10 - \sqrt{100 - 8k}}{4} = 3 \right)</math></p> <p><math>k = 8</math></p> <p><math>x_P = 2, x_Q = 8</math></p> | <p><b>M1*</b></p> <p><b>A1</b></p> <p><b>M1dep*</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>[7]</b></p> | <p><b>2.1</b></p> <p><b>1.1</b></p> <p><b>3.1a</b></p> <p><b>1.1</b></p> <p><b>1.1</b></p> <p><b>2.2a</b></p> | <p>Either <math>\frac{d}{dx}(2xy) = 2y + 2x \frac{dy}{dx}</math> <b>or</b><br/> <math>\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}</math></p> <p>Correct equation (any form) – allow = 0 implied by later working</p> <p>Set the denominator of their <math>\frac{dy}{dx}</math> equal to zero (oe)</p> <p>Eliminate <math>x</math> correctly and simplify to a 3TQ quadratic in <math>y</math> (if correct: <math>2y^2 - 10y + k = 0</math>) <b>or</b> eliminate <math>y</math> correctly to form a 3TQ quadratic in <math>x</math> (if correct: <math>x^2 - 10x + 2k = 0</math>)</p> <p>Setting up the equation for diff. in <math>y</math><br/> <math>\frac{\sqrt{b^2 - 4ac}}{a} = \pm 3</math>, LHS of equation must be correct following through from their 3TQ (the constant term for their 3TQ in <math>y</math> must contain <math>k</math>)</p> <p>www (can be implied by correct <math>y</math>-values)</p> <p>Both correct www – do not penalise <math>x_P = 8, x_Q = 2</math></p> | <p>Attempt at implicit differentiation wrt <math>x</math></p> <p>Setting derivative equal to <math>y'</math> is <b>A0</b></p> <p>Denominator must be linear in both <math>x</math> and <math>y</math></p> <p>Dependent on both previous <b>M</b> marks – condone sign slips only in simplification</p> <p>Dependent on first two <b>M</b> marks – if considering diff. in <math>x</math> values = 3 then <b>M0</b></p> <p><b>Correct <math>k</math> or both <math>y</math>'s www is M1 A1</b></p> <p><math>y_P = 1, y_Q = 4</math></p> <p><b>Dependent on all previous marks</b></p> |

| Question |  |  | Answer                                                                | Marks  | AO | Guidance                                                                                                                                          |                                                                                                                    |
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|          |  |  | Alternative for the first 3 marks                                     |        |    | Implicit differentiation wrt y                                                                                                                    |                                                                                                                    |
|          |  |  | Attempt at implicit differentiation wrt y                             | M1     |    | Either $\frac{d}{dx}(2xy) = 2x + 2y \frac{dx}{dy}$ <b>or</b><br>$\frac{d}{dy}(5x) = 5 \frac{dx}{dy}$                                              |                                                                                                                    |
|          |  |  | $5 \frac{dx}{dy} - 2x - 2y \frac{dx}{dy} + 4y = 0$                    | A1     |    | Correct equation (any form) – allow = 0 implied by later working                                                                                  | Setting derivative equal to x' is A0                                                                               |
|          |  |  | $\frac{dx}{dy} = \frac{4y - 2x}{2y - 5}$ so at P and Q, $4y - 2x = 0$ | M1     |    | Set the numerator of their $\frac{dx}{dy} = 0$ oe                                                                                                 |                                                                                                                    |
|          |  |  | Alternative for the first 4 marks                                     |        |    | Making x the subject, diff. wrt y                                                                                                                 |                                                                                                                    |
|          |  |  | $5x - 2xy + 2y^2 - k = 0 \Rightarrow x = \frac{k - 2y^2}{5 - 2y}$     | M1*    |    | Makes x the subject – must be of the form $x = \frac{f(y)}{g(y)}$                                                                                 | where f(y) is a two-term quadratic fn. in y and g(y) is a two-term linear fn. in y                                 |
|          |  |  | $\frac{dx}{dy} = \frac{(5 - 2y)(-4y) - (k - 2y^2)(-2)}{(5 - 2y)^2}$   | A1     |    | Correct derivative (allow unsimplified)                                                                                                           | Any equivalent correct form                                                                                        |
|          |  |  | $(5 - 2y)(-4y) - (k - 2y^2)(-2) = 0$                                  | M1dep* |    | Set the numerator of their $\frac{dx}{dy}$ equal to zero (oe) where<br>$\frac{dx}{dy} = \frac{g(y) \times f'(y) \pm f(y) \times g'(y)}{(g(y))^2}$ | Where f(y) and g(y) are as defined in the first M mark and derivatives follow through correctly from their f and g |
|          |  |  | $-20y + 8y^2 + 2k - 4y^2 = 0$<br>$\Rightarrow 2y^2 - 10y + k = 0$     | M1     |    | Simplify to a 3TQ quadratic in y (if correct: $2y^2 - 10y + k = 0$ )                                                                              | Dependent on both previous M marks – condone sign slips only in simplification to 3 TQ in y                        |

| Question |  |  | Answer                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Marks     | AO | Guidance                                                                                                                                                                                                                                                                                                                                           |                                                                                                         |
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|          |  |  | <b>Alternative 1 for the fourth M mark</b>                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |    | <b>Roots of polynomials</b>                                                                                                                                                                                                                                                                                                                        |                                                                                                         |
|          |  |  | <p>If correct: <math>2y^2 - 10y + k [= 0]</math><br/>           Difference in <math>y</math> values is 3 therefore if the <math>y</math> roots are <math>\alpha, 3 + \alpha</math> then <math>\alpha + (3 + \alpha) = -\frac{-10}{2}</math><br/> <math>(\alpha = 1 \Rightarrow y = 1, 4)</math></p> <p><b>If using this method, then candidates do not need to find <math>k</math>, so the penultimate A mark is for the correct two <math>y</math> values (1 and 4)</b></p> | <b>M1</b> |    | <p>Using <math>\sum \alpha = -\frac{b}{a}</math> correctly for their 3TQ in <math>y</math> with roots that differ by 3</p>                                                                                                                                                                                                                         | <p>Could use e.g. <math>\alpha, \alpha - 3</math> etc.</p> <p>Dependent on first two <b>M</b> marks</p> |
|          |  |  | <b>Alternative 2 for the fourth M mark</b>                                                                                                                                                                                                                                                                                                                                                                                                                                   |           |    | <b>Difference in x-values</b>                                                                                                                                                                                                                                                                                                                      |                                                                                                         |
|          |  |  | <p>Difference in <math>y</math>-values is 3 therefore the difference in <math>x</math> values is 6 (from <math>4y - 2x = 0</math>)<br/>           Therefore, as <math>x^2 - 10x + 2k = 0</math><br/> <math>\Rightarrow \sqrt{100 - 8k} = 6</math><br/> <math>\left( \text{or } \frac{10 + \sqrt{100 - 8k}}{2} - \frac{10 - \sqrt{100 - 8k}}{2} = 6 \right)</math></p>                                                                                                        | <b>M1</b> |    | <p>Setting up the equation for diff. in <math>x</math><br/> <math>\frac{\sqrt{b^2 - 4ac}}{a} = \pm \text{their } 6</math>, with their 6<br/>           from their <math>4(3) - 2x = 0</math>, LHS of equation must be correct following through from their 3TQ (the constant term for their 3TQ in <math>x</math> must contain <math>k</math>)</p> | <p>Dependent on first two <b>M</b> marks</p>                                                            |