

- 10** In a certain region, the populations of grey squirrels,  $P_G$  and red squirrels  $P_R$ , at time  $t$  years are modelled by the equations:

$$P_G = 10\,000(1 - e^{-kt})$$

$$P_R = 20\,000e^{-kt}$$

where  $t \geq 0$  and  $k$  is a positive constant.

- (a)** **(i)** On the axes in your Printed Answer Book, sketch the graphs of  $P_G$  and  $P_R$  on the same axes.
- (ii)** Give the equations of any asymptotes. **[4]**
- (b)** What does the model predict about the long term population of
- grey squirrels
  - red squirrels? **[2]**
- Grey squirrels and red squirrels compete for food and space. Grey squirrels are larger and more successful than red squirrels.
- (c)** Comment on the validity of the model given by the equations, giving a reason for your answer. **[1]**
- (d)** Show that, according to the model, the rate of decrease of the population of red squirrels is always double the rate of increase of the population of grey squirrels. **[4]**
- (e)** When  $t = 3$ , the numbers of grey and red squirrels are equal. Find the value of  $k$ . **[4]**