

Question	Answer	Marks	AOs	Guidance
10	<p>Curve crosses the x-axis when $y = 0$</p> $y = (k - x) \ln x = 0$ <p>Either $k - x = 0$ or $\ln x = 0$ $x = k$ or 1</p> <p>EITHER</p> $\text{Area} = \int_1^k (k - x) \ln x \, dx$ <p>Let $u = \ln x$, $\frac{dv}{dx} = k - x$, $\frac{du}{dx} = \frac{1}{x}$, $v = kx - \frac{1}{2}x^2$</p> $\text{Area} = \left[\left(kx - \frac{1}{2}x^2 \right) \ln x \right]_1^k - \int_1^k \frac{1}{x} \left(kx - \frac{1}{2}x^2 \right) dx$ $\left[\left(kx - \frac{1}{2}x^2 \right) \ln x \right]_1^k - \int_1^k \left(k - \frac{1}{2}x \right) dx$ $\left[\left(kx - \frac{1}{2}x^2 \right) \ln x - \left(kx - \frac{1}{4}x^2 \right) \right]_1^k$ $\left(\left(k^2 - \frac{1}{2}k^2 \right) \ln k - \left(k^2 - \frac{1}{4}k^2 \right) \right) - \left(\left(k - \frac{1}{2} \right) \ln 1 - \left(k - \frac{1}{4} \right) \right)$ $= \frac{1}{2}k^2 \ln k - \frac{3}{4}k^2 + k - \frac{1}{4}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1dep</p> <p>A1</p> <p>[8]</p>	<p>3.1a</p> <p>1.1b</p> <p>2.1</p> <p>1.1b</p> <p>3.1a</p> <p>1.1b</p> <p>1.1a</p> <p>1.1b</p>	<p>Attempt to solve $y = 0$</p> <p>Both roots required</p> <p>Using integration by parts with $u = \ln x$, $\frac{dv}{dx} = k - x$ clearly argued</p> <p>Allow without limits</p> <p>Simplifying the integrand</p> <p>Second part correct</p> <p>Using limits. Dependent on M mark for integration by parts</p> <p>Cao</p>

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10	<p>OR Integral split into two separate integrals</p> $\int_1^k k \ln x \, dx$ <p>Let $u = \ln x$, $\frac{dv}{dx} = k$, $\frac{du}{dx} = \frac{1}{x}$, $v = kx$</p> $= [kx \ln x]_1^k - \int_1^k \frac{1}{x} kx \, dx$ $[kx \ln x]_1^k - \int_1^k k \, dx = [kx \ln x - kx]_1^k$ $(k^2 \ln k - k^2) - (k \ln 1 - k) = k^2 \ln k - k^2 + k$ <p>And</p> $\text{Area} = \int_1^k x \ln x \, dx$ <p>Let $u = \ln x$, $\frac{dv}{dx} = x$, $\frac{du}{dx} = \frac{1}{x}$, $v = \frac{1}{2}x^2$</p> $= \left[\frac{1}{2}x^2 \ln x \right]_1^k - \int_1^k \frac{1}{x} \times \frac{1}{2}x^2 \, dx$ $\left[\frac{1}{2}x^2 \ln x \right]_1^k - \int_1^k \frac{1}{2}x \, dx$ $\left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 \right]_1^k$ $\left(\frac{1}{2}k^2 \ln k - \frac{1}{4}k^2 \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right) = \frac{1}{2}k^2 \ln k - \frac{1}{4}k^2 + \frac{1}{4}$ $\text{Area} = (k^2 \ln k - k^2 + k) - \left(\frac{1}{2}k^2 \ln k - \frac{1}{4}k^2 + \frac{1}{4} \right)$ $= \frac{1}{2}k^2 \ln k - \frac{3}{4}k^2 + k - \frac{1}{4}$	<p>M1</p> <p>M1</p> <p>M1dep</p> <p>A1</p> <p>A1</p> <p>A1</p>		<p>Using integration by parts with $u = \ln x$, $\frac{dv}{dx} = k$</p> <p>or $u = \ln x$, $\frac{dv}{dx} = \pm x$ clearly argued</p> <p>Simplifying the integrand</p> <p>Substitution of limits seen in at least one integral. Dependent on M mark for integration by parts</p> <p>Both integrals correct at this stage Allow without limits</p> <p>Both integrals fully correct Allow without limits</p> <p>Cao</p>