

- 13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27-8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	-0.056
0.5	2.9625	-0.228
0.75	2.8694	-0.547
1	2.6684	-1.124
1.25	2.2490	-1.977
1.5	0	ERROR

- (i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. [3]
- (ii) Find the first three terms of the binomial expansion of $f(x)$. [3]
- (iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3} dx$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) [2]
- (iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3} dx$. [3]

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3} dx$. The graph of $y = f(x)$ is shown in Fig. 13.

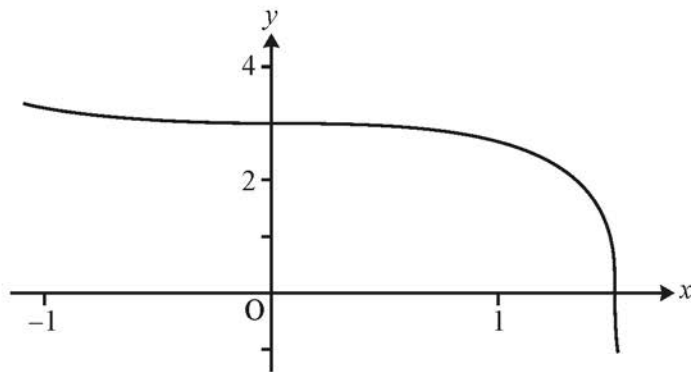


Fig. 13

- (v) Explain why the trapezium rule gives an underestimate. [1]