| Question |  | Answer | Marks | AOs | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | (a) | $\frac{\mathrm{d} V}{\mathrm{~d} t}=\frac{k}{x}$ <br> When $t=0, x=5$ and $\frac{\mathrm{d} V}{\mathrm{~d} t}=21$, so $\frac{\mathrm{d} V}{\mathrm{~d} t}=\frac{105}{x}$ $\frac{\mathrm{d} V}{\mathrm{~d} x}=4 \pi x^{2}$ <br> so the chain rule gives $\frac{\mathrm{d} V}{\mathrm{~d} t}=\frac{\mathrm{d} V}{\mathrm{~d} x} \times \frac{\mathrm{d} x}{\mathrm{~d} t}=4 \pi x^{2} \frac{\mathrm{~d} x}{\mathrm{~d} t}$ <br> Hence $\frac{\mathrm{d} x}{\mathrm{~d} t}=\frac{1}{4 \pi x^{2}} \times \frac{105}{x}=\frac{105}{4 \pi x^{3}}$ <br> AG | M1 <br> A1 <br> B1 <br> M1 <br> A1 <br> [5] | 3.3 <br> 3.3 <br> 2.1 <br> 2.1 <br> 3.3 | Expresses inverse proportionality with a constant <br> Evaluating $k$, oe (may be done later) <br> may be embedded in chain rule <br> Use of the chain rule <br> Convincing argument |  |
| 11 | (b) | $\begin{aligned} & \int 4 \pi x^{3} \mathrm{~d} x=\int 105 \mathrm{~d} t \\ & \pi x^{4}=105 t+c \end{aligned}$ <br> When $t=0, x=5$ so $c=625 \pi$ <br> When $t=120 \quad x=\sqrt[4]{\frac{105}{\pi} \times 120+625}=8.25 \mathrm{~cm}$ | $\begin{gathered} \text { M1 } \\ \text { A1 } \\ \text { M1 } \\ \text { A1 } \\ \text { A1 } \\ {[5]} \end{gathered}$ | $\begin{gathered} \hline \text { 3.1a } \\ \text { 1.1b } \\ 3.3 \\ 3.3 \\ \\ \hline 3.4 \end{gathered}$ | Separating the variables <br> Condone missing $+c$ here <br> Using initial conditions Correct value for $c$ <br> cao |  |
| 11 | (c) | As $t$ gets very large, the volume gets very large so the balloon will get beyond the maximum it can be without bursting and so burst. | E1 <br> [1] | 3.5b | Conveys the idea that $t \rightarrow \infty \Rightarrow V \rightarrow \infty \text { or } x \rightarrow \infty$ <br> Indicates a practical problem with very large volume |  |

