Kareem wants to solve the equation $\sin 4x + e^{-x} + 0.75 = 0$. He uses his calculator to create the following table of values for $f(x) = \sin 4x + e^{-x} + 0.75$.

x	0	1	2	3	4	5	6
f(x)	1.750	0.361	1.875	0.263	0.480	1.670	-0.153

He argues that because f(6) is the first negative value in the table, there is a root of the equation between 5 and 6.

[1]

[1]

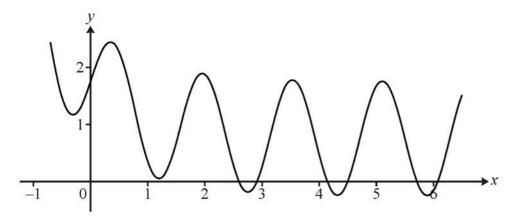
[2]

[1]

(a) Comment on the validity of his argument.

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The diagram shows the graph of $y = \sin 4x + e^{-x} + 0.75$.



(b) Explain why Kareem failed to find other roots between 0 and 6.

Kareem decides to use the Newton-Raphson method to find the root close to 3.

- (c) (i) Determine the iterative formula he should use for this equation.
 - (ii) Use the Newton-Raphson method with $x_0 = 3$ to find a root of the equation f(x) = 0. Show **three** iterations and give your answer to a suitable degree of accuracy. [3]

Kareem uses the Newton-Raphson method with $x_0 = 5$ and also with $x_0 = 6$ to try to find the root which lies between 5 and 6. He produces the following tables.

x_0	5		
x_1	3.97288		
x_2	4.12125		

x_0	6		
x_1	6.09036		
x_2	6.07110		

- (d) (i) For the iteration beginning with $x_0 = 5$, represent the process on the graph in the Printed Answer Booklet. [2]
 - (ii) Explain why the method has failed to find the root which lies between 5 and 6. [2]
 - (iii) Explain how Kareem can adapt his method to find the root between 5 and 6.