| Question |  | Answer | Marks | AO | Guidance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 | (a) | When $t=0, \quad 82=\theta_{0} \mathrm{e}^{0}$ so $\theta_{0}=82$ $\begin{aligned} & t=5, \quad 27=\theta_{0} \mathrm{e}^{-5 k} \\ & \text { giving } k=\left[-\frac{1}{5} \ln \left(\frac{27}{82}\right)\right]=0.222 \text { to } 3 \mathrm{sf} \end{aligned}$ | B1 M1 A1 [3] | $\begin{gathered} \hline 3.3 \\ 3.3 \\ 1.1 \mathrm{~b} \end{gathered}$ | Forming an equation for $k$ and attempt to solve <br> Allow for exact value or evaluated to at least 2 s.f. |
| 14 | (b) | The model predicts that temperature tends to zero but if the quantity of water is small the water will warm up so it will not cool the object to zero. | E1 [1] | 3.5b | Must imply to the model tends to zero and this does not match the real situation. |
| 14 | (c) | $\begin{aligned} & \ln \theta=\ln \left(\theta_{0} \mathrm{e}^{-k t}\right)=\ln \theta_{0}+\ln \left(\mathrm{e}^{-k t}\right) \\ & \ln \theta=\ln 82-0.222 t=[4.41-0.222 t] \end{aligned}$ | M1 <br> A1 <br> [2] | $2.1$ $2.1$ | Taking logs and attempting to use laws of logs <br> Do not award for values of $a$ and $b$ obtained directly from the data and the natural log form of the model. <br> FT their values for $\theta_{0}$ and $k$ <br> Accept as part of equation or $a$ and $b$ clearly stated |
| 14 | (d) | When $t=0, \ln \theta=3.4$ <br> giving $\theta=29.96$ so $30.0^{\circ} \mathrm{C}$ to 3 sf $\begin{aligned} & \theta=29.96 \mathrm{e}^{-0.08 t} \\ & \frac{\mathrm{~d} \theta}{\mathrm{~d} t}=29.96 \times-0.08 \mathrm{e}^{-0.08 t} \end{aligned}$ <br> When $t=0 \frac{\mathrm{~d} \theta}{\mathrm{~d} t}=-2.3968$ [object is cooling by $2.4^{\circ}$ per minute] | B1 <br> M1 <br> A1 <br> A1 <br> [4] | 3.4 <br> 3.4 <br> 3.4 <br> 3.4 | Accept $30^{\circ}$ www Must be evaluated <br> Attempt to differentiate their exponential expression for $\theta$ <br> Any form eg $\mathrm{e}^{3.4} \times-0.08 \mathrm{e}^{-0.08 t}$ or $-0.08 \mathrm{e}^{3.4-0.08 t}$ <br> Allow for correct negative value for $\frac{\mathrm{d} \theta}{\mathrm{d} t}$ or a clear statement that the rate of cooling is $2.4^{\circ}$ per minute. Accept $=-0.08 \mathrm{e}^{3.4}$ |


|  |  | Alternative method <br> When $t=0, \ln \theta=3.4$ <br> giving $\theta=29.96$ so $30.0^{\circ} \mathrm{C}$ to 3 sf <br> Differentiate $\ln \theta=3.4-0.08 t$ w.r.t $t$ $\begin{aligned} & \frac{1}{\theta} \frac{\mathrm{~d} \theta}{\mathrm{~d} t}=-0.08 \\ & \frac{\mathrm{~d} \theta}{\mathrm{~d} t}=-0.08 \theta \end{aligned}$ <br> When $t=0, \theta=29.96$ $\text { so } \frac{\mathrm{d} \theta}{\mathrm{~d} t}=-2.3968$ <br> object is cooling by $2.4^{\circ}$ per minute | B1 <br> M1 <br> A1 <br> A1 <br> [4] | 3.4 | Accept $30^{\circ}$ www <br> Uses implicit differentiation w.r.t $t$ <br> Correct derivative <br> Allow for correct negative value for $\frac{\mathrm{d} \theta}{\mathrm{d} t}$ the rate of cooling is $2.4^{\circ}$ per minute | or a clear statement that |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14 | (e) | Solve simultaneously <br> $\ln \theta=3.4-0.08 t$ <br> $\ln \theta=\ln 82-0.222 t$ <br> gives $t=7.089 \quad t=7.1[7$ minutes and 5 <br> seconds] $\ln \theta=2.8328 \text { gives } \theta=17^{\circ} \mathrm{C}$ <br> Alternative method <br> $82 \mathrm{e}^{-0.222 t}=30 \mathrm{e}^{-0.08 t}$ <br> $\frac{82}{30}=\mathrm{e}^{0.142 t}$ <br> $t=7.08$ [7 minutes and 5 seconds] <br> $\theta=17^{\circ} \mathrm{C}$ | M1 <br> A1 <br> A1 <br> M1 <br> A1 <br> A1 <br> [3] | 3.1b <br> 3.4 <br> 3.4 | Attempting to find the intersection of their (c) and the given line <br> Accept awrt 7.0, 7.1 or 7.2 <br> Must be the value for $\theta$ <br> Equate their expressions for temperature and attempts to solve for $t$ Accept awrt 7.0, 7.1 or 7.2 Cao | Could be BC |

