14 Alex places a hot object into iced water and records the temperature $\theta\,°\text{C}$ of the object every minute. The temperature of an object t minutes after being placed in iced water is modelled by $\theta = \theta_0 e^{-kt}$ where θ_0 and k are constants whose values depend on the characteristics of the object.

The temperature of Alex's object is 82 °C when it is placed into the water. After 5 minutes the temperature is 27 °C.

(a) Find the values of θ_0 and k that best model the data. **[3]**

(b) Explain why the model may **not** be suitable in the long term if Alex does not top up the ice in the water. **[1]**

(c) Show that the model with the values found in part **(a)** can be written as $\ln\theta = a - bt$ where a and b are constants to be determined. **[2]**

Ben places a different object into iced water at the same time as Alex. The model for Ben's object is $\ln\theta = 3.4 - 0.08t$.

(d) Determine each of the following:

- the initial temperature of Ben's object
- the rate at which Ben's object is cooling initially. **[4]**

(e) According to the models, there is a time at which both objects have the same temperature.

Find this time and the corresponding temperature. **[3]**