**10** **In this question you must show detailed reasoning.**

The equation of a curve is

$$y = \frac{\sin 2x - x}{x \sin x}.$$

(a) Use the small angle approximation given in the list of formulae on pages 2–3 of this question paper to show that

$$\int_{0.01}^{0.05} y \, dx \approx \ln 5.$$

[4]

(b) Use the same small angle approximation to show that

$$\frac{dy}{dx} \approx -10000 \text{ at the point where } x = 0.01.$$

[2]

The equation $y = 0$ has a root near $x = 1$. Joan uses the Newton-Raphson method to find this root. The output from the spreadsheet she uses is shown in Fig. 10.1.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----|---|---|---|---|---|---|---|---|
| $x_n$ | 1 | 0.958509 | 0.950084 | 0.948261 | 0.94786 | 0.947772 | 0.947753 | 0.947748 |

**Fig. 10.1**

Joan carries out some analysis of this output. The results are shown in Fig. 10.2.

| $x$ | $y$ |
|-----|-----|
| 0.9477475 | −7.79967E−07 |
| 0.9477485 | −2.90821E−06 |
| $x$ | $y$ |
| 0.947745 | 4.54066E−06 |
| 0.947755 | −1.67417E−05 |

**Fig. 10.2**

(c) Consider the information in Fig. 10.1 and Fig. 10.2.

- Write 4.54066E–06 in standard mathematical notation.
- State the value of the root as accurately as you can, justifying your answer. [3]