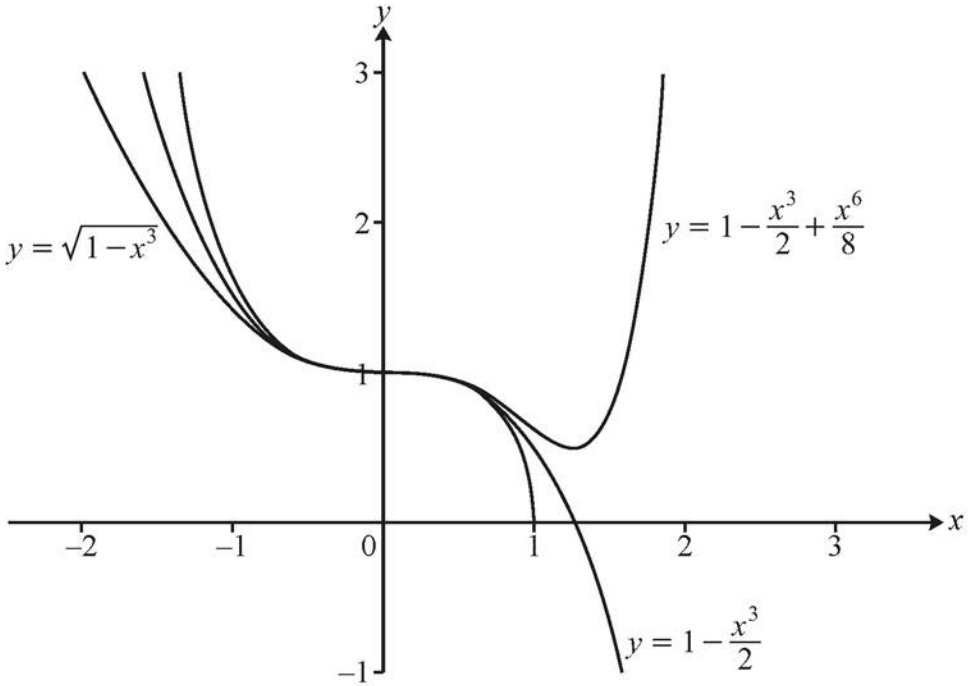
7 A student is trying to find the binomial expansion of $\sqrt{1-x^3}$.

She gets the first three terms as $1-\frac{x^3}{2}+\frac{x^6}{8}$.

She draws the graphs of the curves $y=\sqrt{1-x^3}$, $y=1-\frac{x^3}{2}$ and $y=1-\frac{x^3}{2}+\frac{x^6}{8}$ using software.



(a) Explain why $1-\frac{x^3}{2}+\frac{x^6}{8} \geqslant 1-\frac{x^3}{2}$ for all values of x. **[1]**

(b) Explain why the graphs suggest that the student has made a mistake in the binomial expansion. **[1]**

(c) Find the first **four** terms in the binomial expansion of $\sqrt{1-x^3}$. **[3]**

(d) State the set of values of x for which the binomial expansion in part **(c)** is valid. **[1]**

(e) Sketch the curve $y=2.5\sqrt{1-x^3}$ on the grid in the Printed Answer Booklet. **[2]**

(f) In this question you must show detailed reasoning.

The end of a bus shelter is modelled by the area between the curve $y=2.5\sqrt{1-x^3}$, the lines $x=-0.75$, $x=0.75$ and the x-axis. Lengths are in metres.

Calculate, using your answer to part **(c)**, an approximation for the area of the end of the bus shelter as given by this model. **[4]**