Question			Answer	Marks	AO	Guidance
5	(a)		$P = 9000 \times \frac{2^0}{299 + 2^0}$	M1	3.4	• Substitutes $t = 0$ into equation • May be implied by correct answer
			30	A1	1.1	• cao, nfww
				[2]		
5	(b)		**DR** $7000 < 9000 \times \frac{2^t}{299 + 2^t}$	B1	3.4	• Substitutes $P = 7000$ and sets up correct inequality • Allow correct use of $\leq$ instead of $<$ • Allow any correct inequality e.g. $9 \times \frac{2^t}{299+2^t} > 7$ or $7000 \leq 9000 \times \frac{2^t}{299+2^t}$
			$2093 + 7 \times 2^t < 9 \times 2^t$ $2^t > 1046.5$	M1*	1.1	• Attempt to simplify to a two-term inequality in 2^t • OR $2^{t+1} > 2093$ • Condone any inequality symbol or = for both M marks
			$t \log 2 > \log 1046.5$ OR $t > \log_2 1046.5$ $t > 10.0313566...$	M1dep	2.1	• Appropriate use of logs to obtain value of t (10.03...) • OR $(t + 1) \log 2 > \log 2093$ • OR $t + 1 > \log_2 2093$
			2010 cao	A1	3.2a	• Answer given as a year
				[4]		

Question			Answer	Marks	AO	Guidance
5	(c)		$\frac{dP}{dt} = [9000\times]\frac{(299+2^t)2^t\ln(2) - 2^t \times 2^t\ln(2)}{(299+2^t)^2}$	M1	3.1b	• Use of quotient rule to differentiate • Condone errors in derivative of 2^t provided quotient rule in form $\frac{vu'-uv'}{v^2}$ (v, u and denominator must be correct, and numerator correct way round with a subtraction, condone errors in u' and v') • Allow use of product rule in form $vu' + uv'$ (v, u correct and addition seen) • Mark clear intent so ignore '9000' for M1 – even if incorrect in numerator • M0 A0 for only subbing in values of t
				A1	2.1	• Correct derivative • Allow omission of 9000
			$\frac{dP}{dt} = 9000\times\frac{299\times 2^t\ln(2)}{(299+2^t)^2}$	A1	1.1	• Simplifying numerator to e.g. $2691000\times 2^t\ln(2)$ • Allow working with numerator and denominator separately • Allow omission of 9000
			Numerator and denominator of fraction both positive so gradient is positive. For the population to decrease, the gradient would have to be negative so it never decreases	A1	2.4	• **nfww, AG** • Conclusion that gradient is positive (or function is increasing) from clear and correct argument and relating gradient to effect on population. • Allow e.g. $\frac{dP}{dx}$ is positive so P is increasing • Not just e.g. $\frac{dP}{dx}$ is positive, must comment on population • A0 for $\frac{dP}{dt}$ is increasing • **SC1** after 0 marks: attempted argument using limits e.g. 2^t is increasing **and** $299+2^t$ is increasing **and** $\frac{2^t}{299+2^t}$ tends to 1 **and** so P increases

Question			Answer	Marks	AO	Guidance
			Alternative method 1 $P = 9000 \times \dfrac{1}{299 \times 2^{-t} + 1}$	**M1**		• Dividing numerator and denominator by 2^t • Condone one arithmetic slip provided intention is clear • Condone omission of 9000
				A1		• All correct – condone omission of 9000
			2^{-t} is (positive and) decreasing as t increases so the denominator is (positive and) decreasing as t increases	**A1**		• Explanation of denominator decreasing
			So P must increase as t increases so P never decreases (as t increases)	**A1**		• **nfww, AG** • Relating denominator to effect on population and clear conclusion in relation to population • Allow e.g. 'population never decreases'
			Alternative method 2 $9000 \times \left(\dfrac{2^{t+1}}{299 + 2^{t+1}} - \dfrac{2^t}{299 + 2^t}\right)$ $= 9000 \times \left(\dfrac{2^{t+1}(299 + 2^t) - 2^t(299 + 2^{t+1})}{(299 + 2^{t+1})(299 + 2^t)}\right)$	**M1**		• Finding $P_{t+1} - P_t$ and writing as a single fraction • Condone omission of 9000 for all M or A marks • Condone one arithmetic slip provided intention is clear • Allow using an inequality e.g. $\left(\frac{2^{t+1}}{299+2^{t+1}}\right) > \frac{2^t}{299+2^t}$ **and** dealing with fractions appropriately e.g. $2^{t+1}(299 + 2^t) > 2^t(299 + 2^{t+1})$
			$9000 \times \left(\dfrac{2^{t+1} \times 299 + 2^{t+1}2^t - 2^t \times 299 - 2^t 2^{t+1}}{(299 + 2^{t+1})(299 + 2^t)}\right)$	**A1**		• Expands brackets in numerator • OR e.g. $2^{t+1} \times 299 + 2^{t+1}2^t > 2^t \times 299 + 2^t 2^{t+1}$
			$= 9000 \times \left(\dfrac{2^t \times 299}{(299 + 2^{t+1})(299 + 2^t)}\right)$	**A1**		• Simplifies numerator to a single term • OR e.g. $2(299 + 2^t) - (299 + 2^{t+1}) > 0$ • OR e.g. $2^{t+1}[\times 299] > 2^t[\times 299]$
			Since $2^t > 0$ (and $299 > 0$) then the numerator and denominator are both positive so $P_{t+1} > P_t$ and the population is increasing.	**A1**		• Clear conclusion in relation to population, **nfww AG** • OR e.g. since $2^t > 0$ then it is shown that $P_{t+1} > P_t$ so population is increasing • Or $2^{t+1} > 2^t$ so P is increasing • Allow e.g. 'population never decreases'

Question			Answer	Marks	AO	Guidance
			Alternative method 3 $P = [9000] \times \dfrac{299 + 2^t - 299}{299 + 2^t}$ $= [9000] \times \left(1 - \dfrac{299}{299 + 2^t}\right)$	**M1**		• Rewrites numerator and simplifies
				A1		• All correct • Condone omission of 9000
			2^t is positive and increasing, so $\frac{299}{299+2^t}$ is decreasing	**A1**		• Explanation of fraction decreasing • Allow $\frac{299}{299+2^t}$ is decreasing as t increases
			So $[9000] \times \left(1 - \frac{299}{299+2^t}\right)$ is always increasing, so the population is always increasing	**A1**		• Clear conclusion in relation to population, **nfww AG**
			Alternative method 4 $\dfrac{P_{t+1}}{P_t} = \dfrac{[9000 \times] \dfrac{2^{t+1}}{299 + 2^{t+1}}}{[9000 \times] \dfrac{2^t}{299 + 2^t}}$	**M1**		• Finding $\frac{P_{t+1}}{P_t}$ and writing as a fraction
			$= \dfrac{2^{t+1}(299 + 2^t)}{2^t(299 + 2^{t+1})}$	**A1**		• Correct fraction
			$\dfrac{P_{t+1}}{P_t} = \dfrac{2(299 + 2^t)}{(299 + 2^{t+1})} = \dfrac{299+(299 + 2^{t+1})}{(299 + 2^{t+1})}$	**A1**		• Rewrites fraction so numerator clearly greater than denominator
			Since $P_{t+1} > P_t$ the population is always increasing	**A1**		• Clear conclusion in relation to population, **nfww AG** • Allow e.g. 'population never decreases'
				[4]		