Summary of key points

4 The graph of $y = \mathrm{f}(x) + a$ is a translation of the graph $y = \mathrm{f}(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$.

5 The graph of $y = \mathrm{f}(x + a)$ is a translation of the graph $y = \mathrm{f}(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$.

6 When you translate a function, any asymptotes are also translated.

7 The graph of $y = a\mathrm{f}(x)$ is a stretch of the graph $y = \mathrm{f}(x)$ by a scale factor of a in the vertical direction.

8 The graph of $y = \mathrm{f}(ax)$ is a stretch of the graph $y = \mathrm{f}(x)$ by a scale factor of $\frac{1}{a}$ in the horizontal direction.

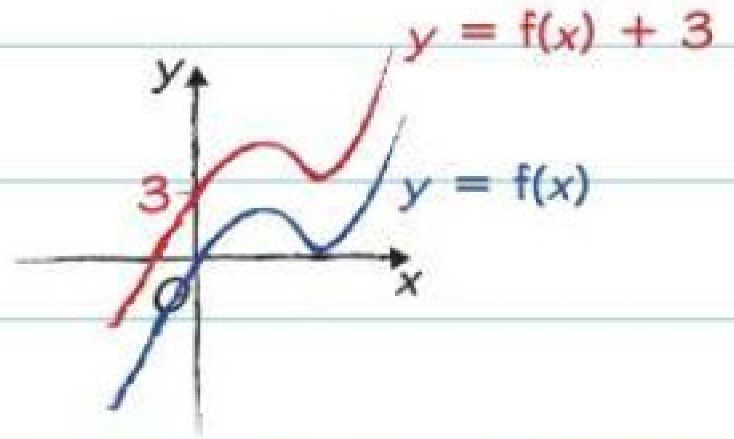
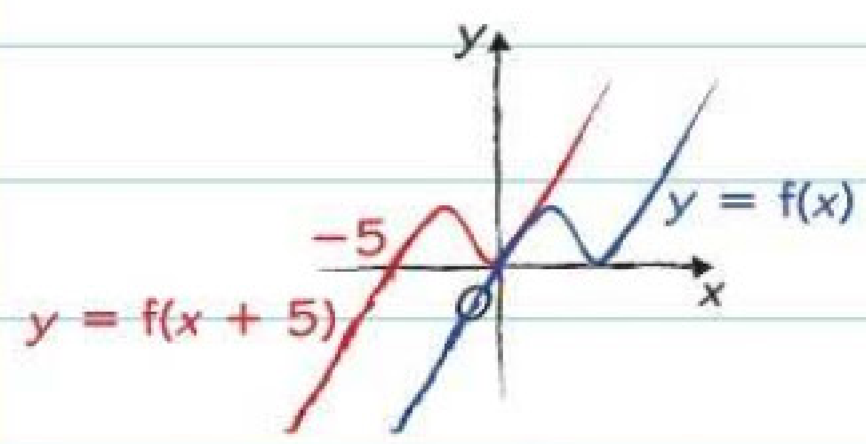
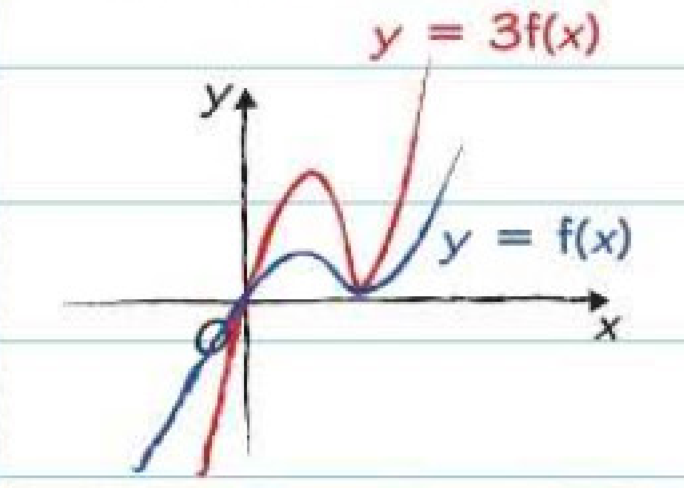
9 The graph of $y = -\mathrm{f}(x)$ is a reflection of the graph of $y = \mathrm{f}(x)$ in the x-axis.

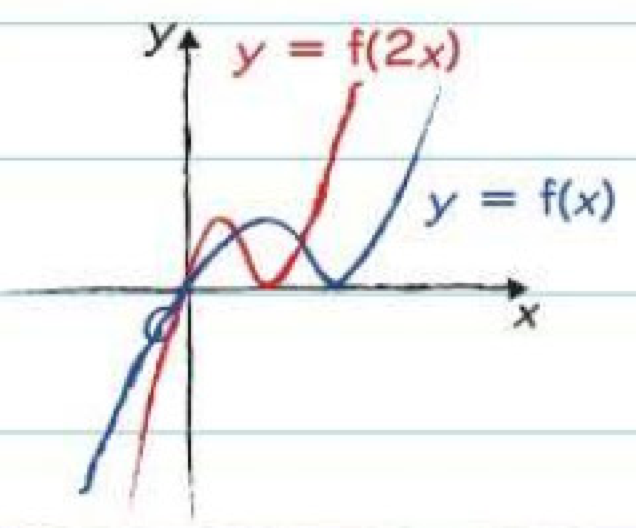
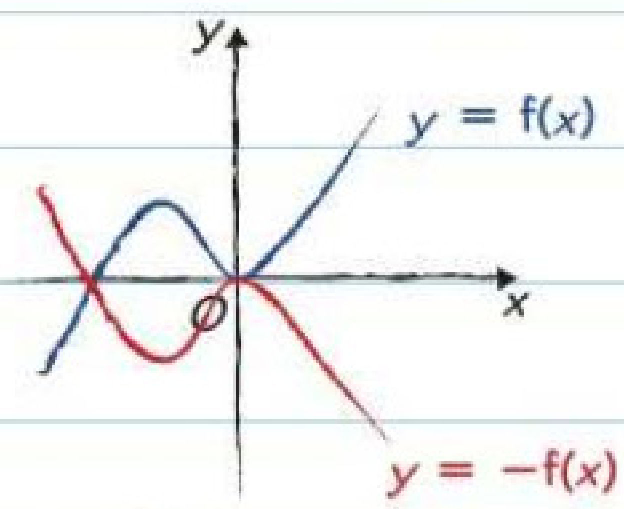
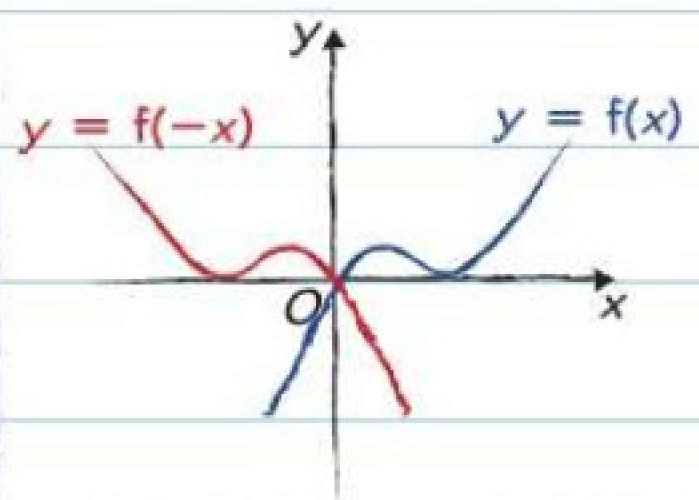
10 The graph of $y = \mathrm{f}(-x)$ is a reflection of the graph of $y = \mathrm{f}(x)$ in the y-axis.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Transformations 1

You can change the equation of a graph to translate it, stretch it or reflect it. These tables show you how you can use functions to transform the graph of $y = f(x)$.

Function	$y = f(x) + a$	$y = f(x + a)$	$y = af(x)$
Transformation of graph	Translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$	Translation $\begin{pmatrix} -a \\ 0 \end{pmatrix}$	Stretch in the vertical direction, scale factor a
Useful to know	$f(x) + a \rightarrow$ move UP a units $f(x) - a \rightarrow$ move DOWN a units	$f(x + a) \rightarrow$ move LEFT a units $f(x - a) \rightarrow$ move RIGHT a units	x-values stay the same
Example			

Function	$y = f(ax)$	$y = -f(x)$	$y = f(-x)$
Transformation of graph	Stretch in the horizontal direction, scale factor $\frac{1}{a}$	Reflection in the x-axis	Reflection in the y-axis
Useful to know	y-values stay the same	'−' outside the bracket	'−' inside the bracket
Example			

Worked example

The diagram shows a sketch of a curve with equation $y = f(x)$.

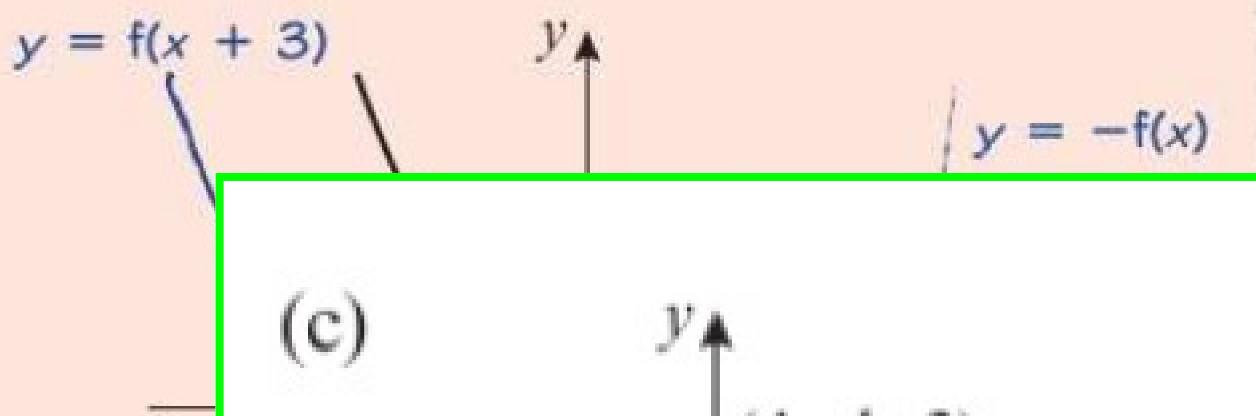
On the same diagram sketch the curve with equation

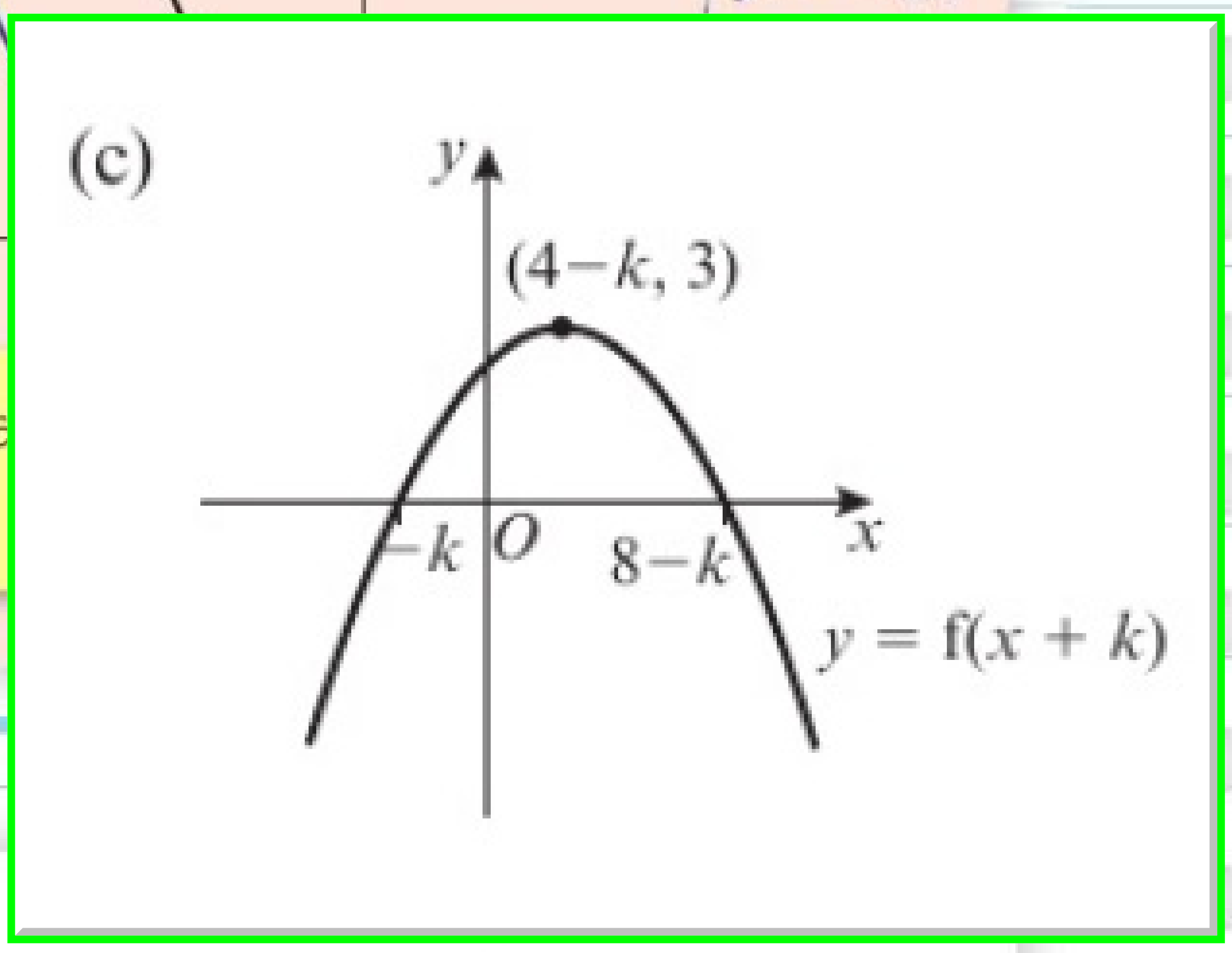
(a) $y = f(x + 3)$ **(3 marks)**

(b) $y = -f(x)$. **(3 marks)**

Show clearly the coordinates of any maximum or minimum points, and any points of intersection with the axes.

Everything in blue is part of the answer.





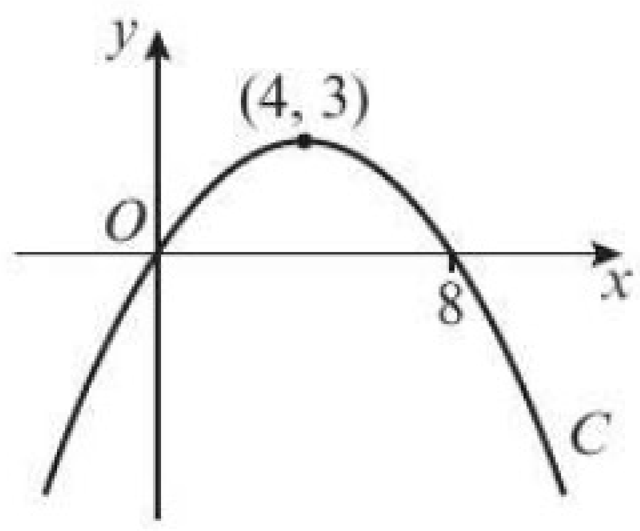
Now try this

The diagram shows a sketch of a curve C with equation $y = f(x)$.
On separate diagrams sketch the curve with equation

(a) $y = 2f(x)$ **(3 marks)** Answer (b) $y = f(-x)$ Answer **(3 marks)**

(c) $y = f(x + k)$, where k is a constant and $0 < k < 4$ Answer **(4 marks)**

On each diagram show the coordinates of any maximum or minimum points, and any points of intersection with the x-axis.



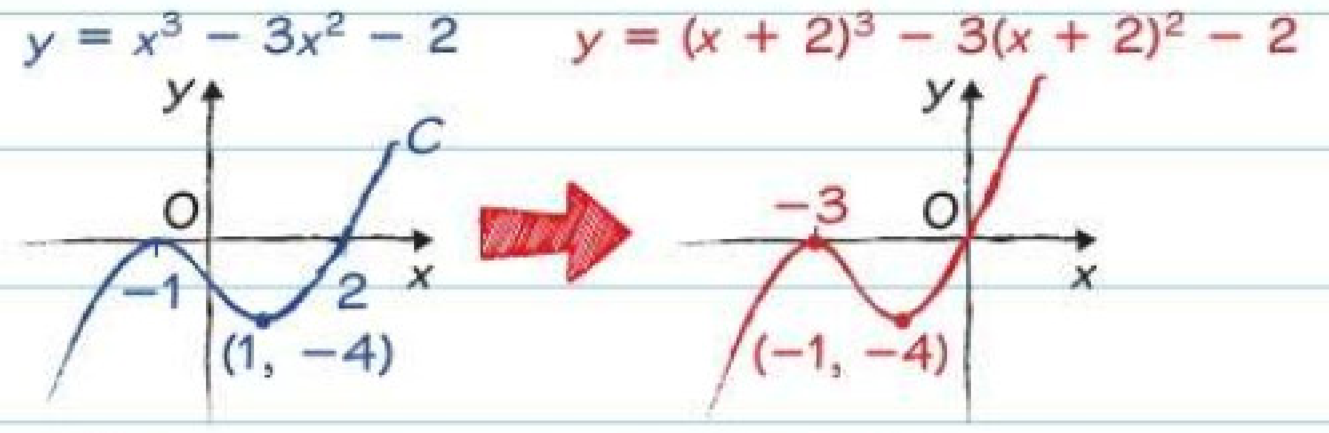
Transformations 2

You need to be able to spot transformed functions from their equations, and sketch transformations involving **asymptotes**.

Functions and equations

Curve C below has equation $y = x^3 - 3x^2 - 2$. You can sketch the curves of other equations by transforming curve C.

$y = x^3 - 3x^2 - 2$ $\quad$ $y = (x + 2)^3 - 3(x + 2)^2 - 2$

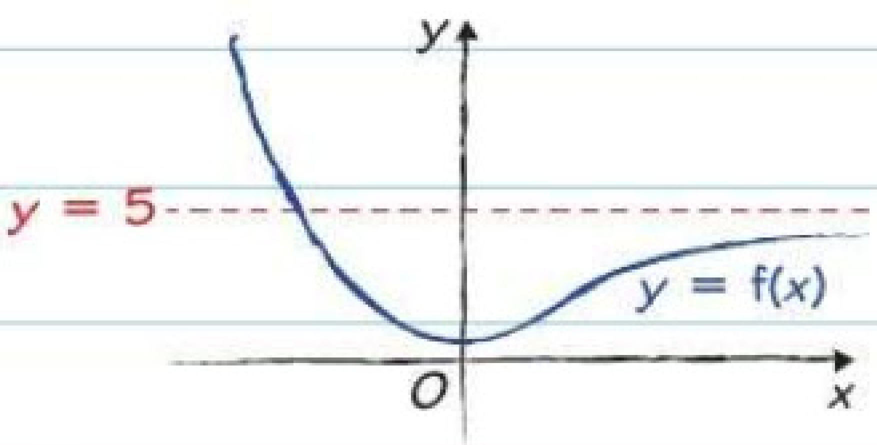


$y = \mathrm{f}(x)$
$y = x^3 - 3x^2 - 2$

$y = \mathrm{f}(x + 2)$
$y = (x + 2)^3 - 3(x + 2)^2 - 2$

Asymptotes

An asymptote is a line which a curve approaches, but never reaches. You draw asymptotes on graphs with DOTTED LINES.



This curve has an asymptote at $y = 5$. When you transform a graph, its asymptotes are transformed as well.

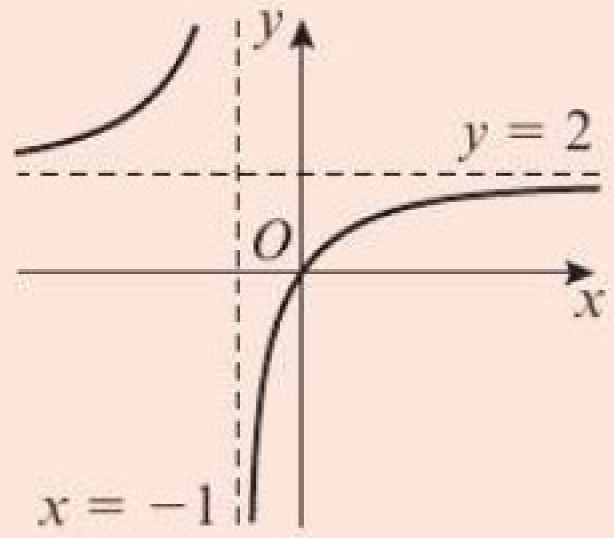
Transformation	New asymptote
$y = \mathrm{f}(x) - 1$	$y = 4$
$y = 2\mathrm{f}(x)$	$y = 10$
$y = \mathrm{f}(x + 4)$	$y = 5$

The graph is translated 4 units to the left, so the horizontal asymptote does not change.

Worked example

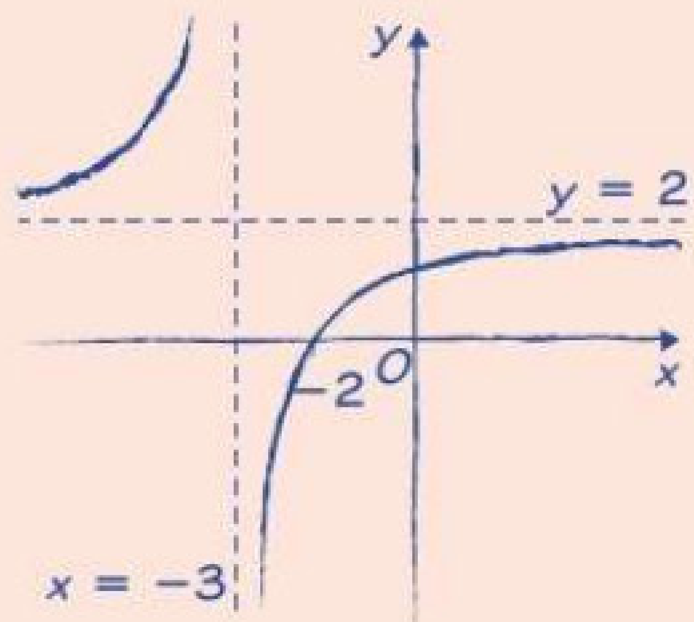
The diagram shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x}{x + 1},\ x \neq -1$$



The curve has asymptotes with equations $y = 2$ and $x = -1$

(a) Sketch the curve with equation $y = \mathrm{f}(x + 2)$ and state the equations of its asymptotes. **(3 marks)**



(b) Find the coordinates of the points where the curve in part (a) crosses the coordinate axes. **(3 marks)**

$$\mathrm{f}(x + 2) = \frac{2(x + 2)}{(x + 2) + 1} = \frac{2x + 4}{x + 3}$$

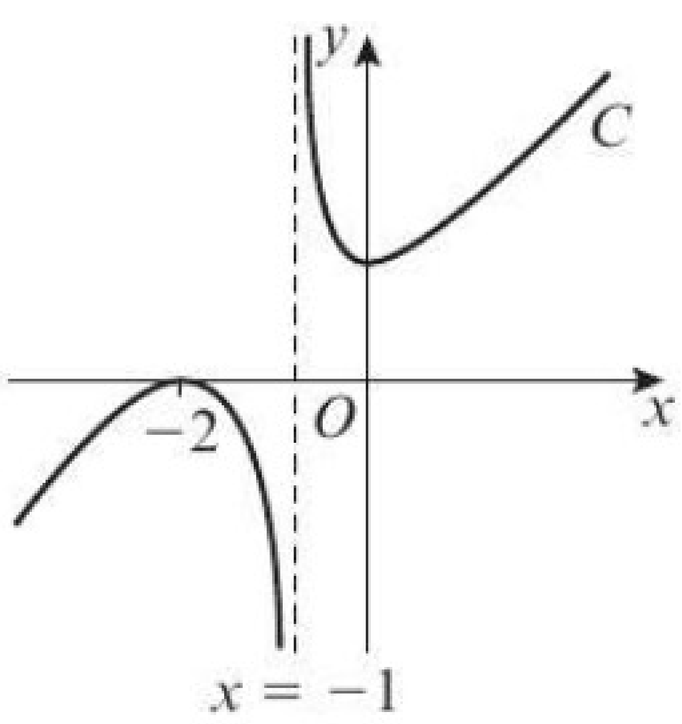
When $x = 0$, $y = \frac{4}{3}$

$(-2, 0)$ and $(0, \frac{4}{3})$

Now try this

The diagram shows a curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{(x + 2)^2}{x + 1},\ x \neq -1$$



(a) Sketch the curve with equation $y = \mathrm{f}(x + 1)$ and state the new equation of the asymptote $x = -1$ Answer **(3 marks)**

(b) Write down the coordinates of the points where the curve meets the coordinate axes. Answer **(3 marks)**