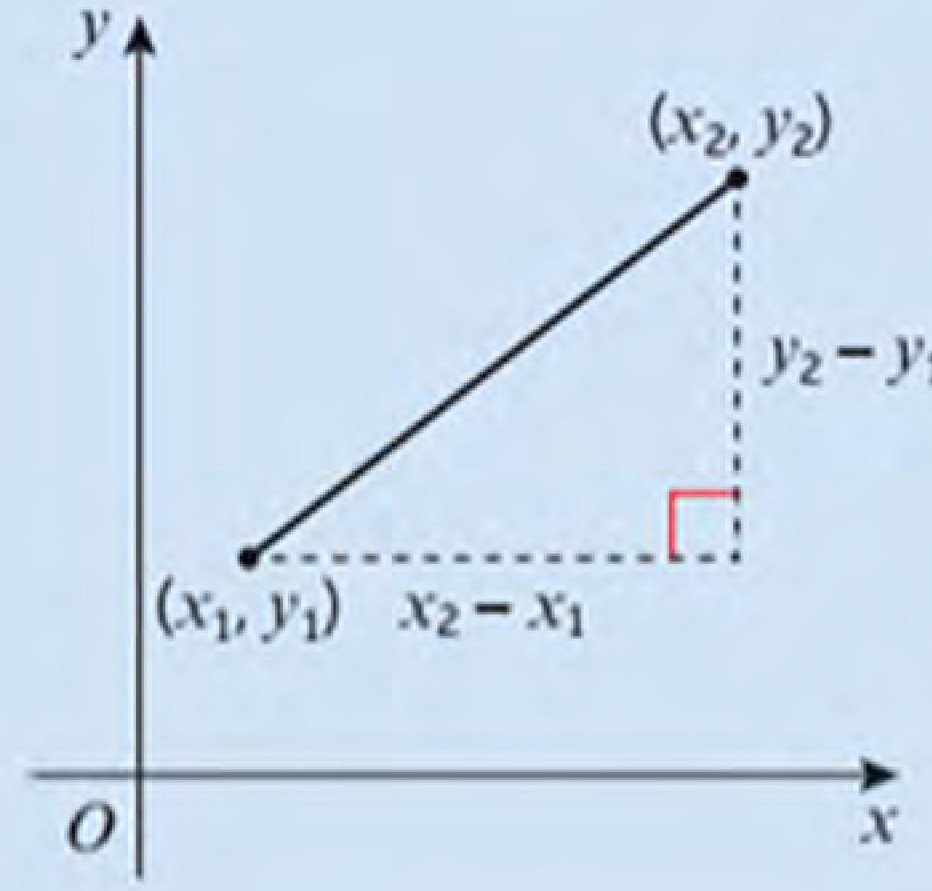
Summary of key points

1 The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



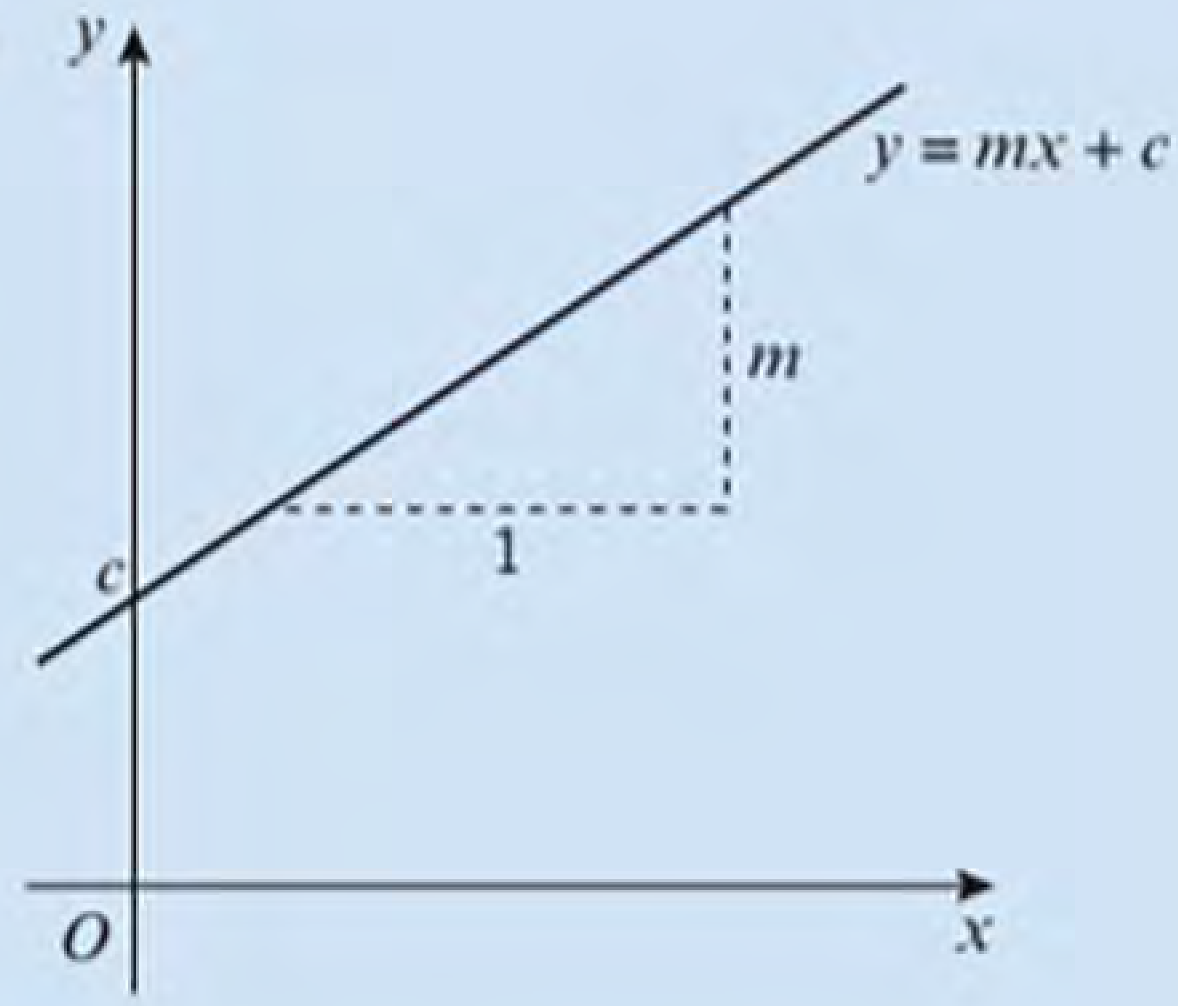
2
- The equation of a straight line can be written in the form

  $y = mx + c$,

  where m is the gradient and $(0, c)$ is the y-intercept.
- The equation of a straight line can also be written in the form

  $ax + by + c = 0$,

  where a, b and c are integers.



3 The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.

4 Parallel lines have the same gradient.

5 If a line has a gradient m, a line perpendicular to it has a gradient of $-\frac{1}{m}$

6 If two lines are perpendicular, the product of their gradients is -1.

7 You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

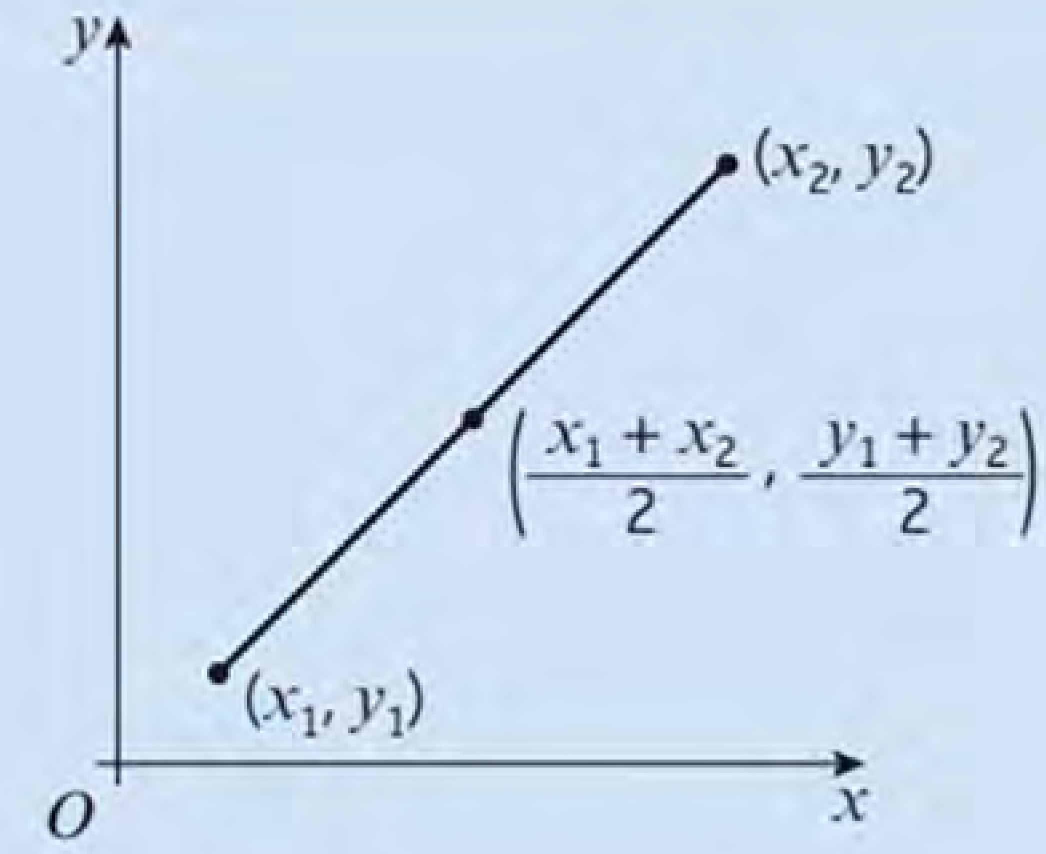
8 The point of intersection of two lines can be found using simultaneous equations.

9 Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.

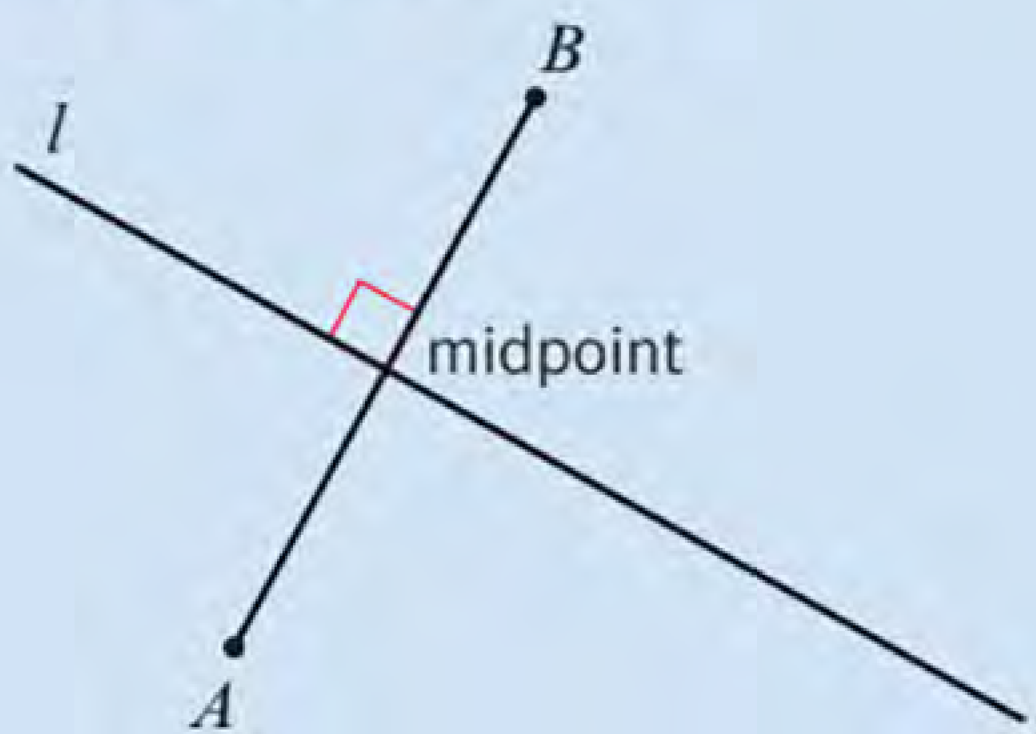
10 A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problems in order to create a model.

Summary of key points

1 The midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.



2 The perpendicular bisector of a line segment AB is the straight line that is perpendicular to AB and passes through the midpoint of AB.



If the gradient of AB is m then the gradient of its perpendicular bisector, l, will be $-\frac{1}{m}$

Had a look ☐ Nearly there ☐ Nailed it! ☐

Equations of lines

The equation of a straight line can be written in the form $y = mx + c$, where m is the **gradient** of the line, and c is the point where it crosses the y-axis. There are other useful ways to write the equation of a straight line.

Point and gradient

If a straight line has **gradient** m and passes through the **point** (x_1, y_1), then you can write its equation as

$$y - y_1 = m(x - x_1)$$

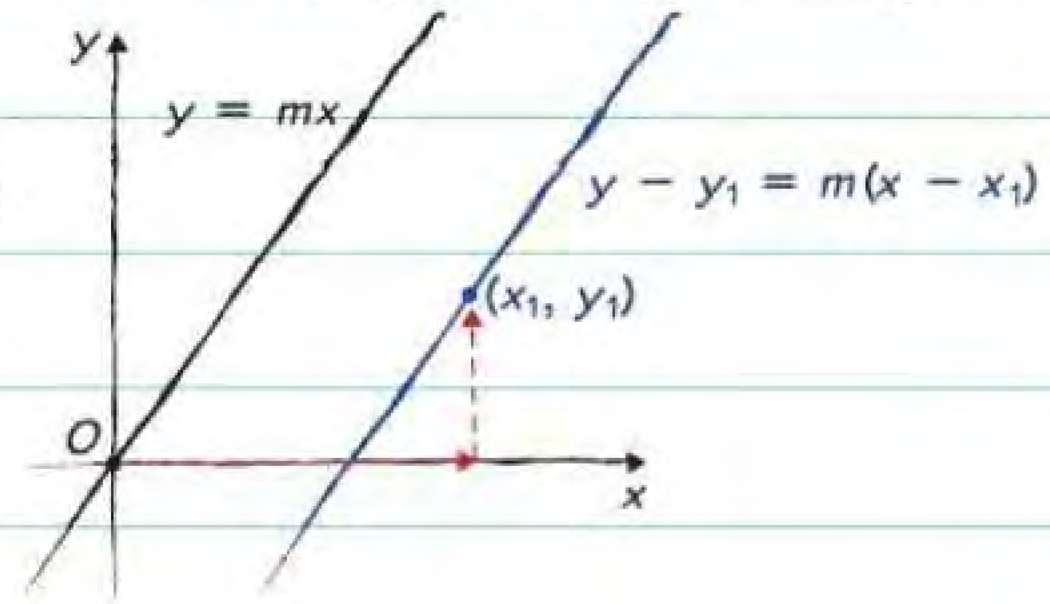
This equation is **very** useful for lots of exam questions! Make sure you learn it.

If you are given two points on a straight line, (x_1, y_1) and (x_2, y_2), you can calculate the gradient using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Thinking in transformations

You can remember this equation by thinking of it as a **translation** of the graph $y = mx$ by vector $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$



Worked example

The line L passes through the point $(-8, 5)$ and has gradient $\frac{1}{2}$. Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \tfrac{1}{2}(x - (-8))$$
$$2y - 10 = x + 8$$
$$x - 2y + 18 = 0$$

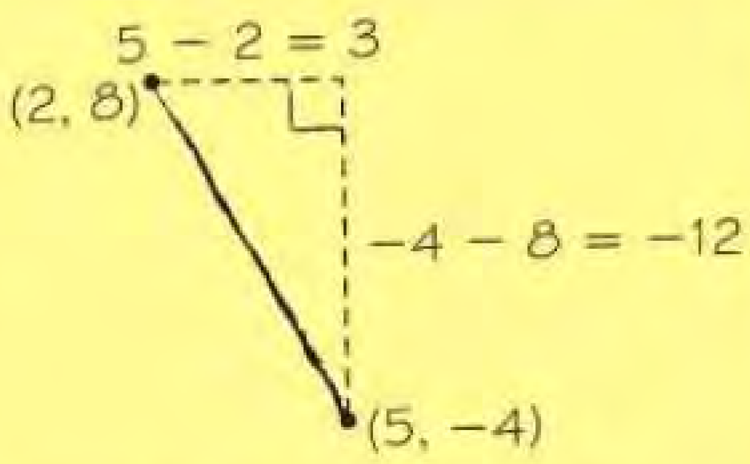
Problem solved!

If you are using a formula which is **not** in the booklet, always **write it down** before you substitute. Here, $m = \frac{1}{2}$, $x_1 = -8$ and $y_1 = 5$.

You need a, b and c to be integers, so multiply every term in your equation by 2 to remove the fraction. Then rearrange so one side is equal to 0.

You will need to use problem-solving skills throughout your exam – **be prepared!**

You can draw a sketch to help you find the gradient, or use $m = \frac{y_2 - y_1}{x_2 - x_1}$



Write the equation as $y = -4x + c$ and substitute the values of x and y from either point on the graph to find c.

Worked example

The line L passes through the points $(2, 8)$ and $(5, -4)$. Find an equation for L in the form $y = mx + c$ **(3 marks)**

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 8}{5 - 2} = \frac{-12}{3} = -4$$
$$y = -4x + c$$
$$8 = -4(2) + c$$
$$c = 16 \quad \text{so} \quad y = -4x + 16$$

Now try this

1. The line L passes through the point $(6, -5)$ and has gradient $-\frac{1}{3}$. Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers. Answer **(3 marks)**
2. The line L passes through $(-4, 2)$ and $(8, 11)$. Find an equation for L in the form $y = mx + c$, where m and c are constants. **(3 marks)**

Answer

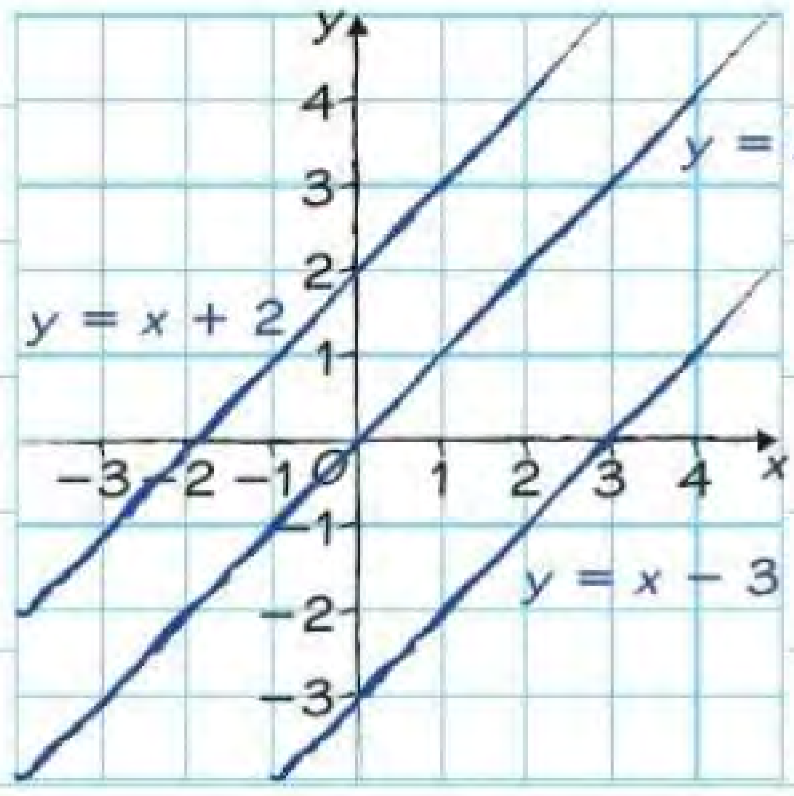
3. The line $3y + 4x - k = 0$ passes through the point $(5, 1)$. Find Answer
   (a) the value of k **(1 mark)**
   (b) the gradient of the line. **(2 marks)**

Answer

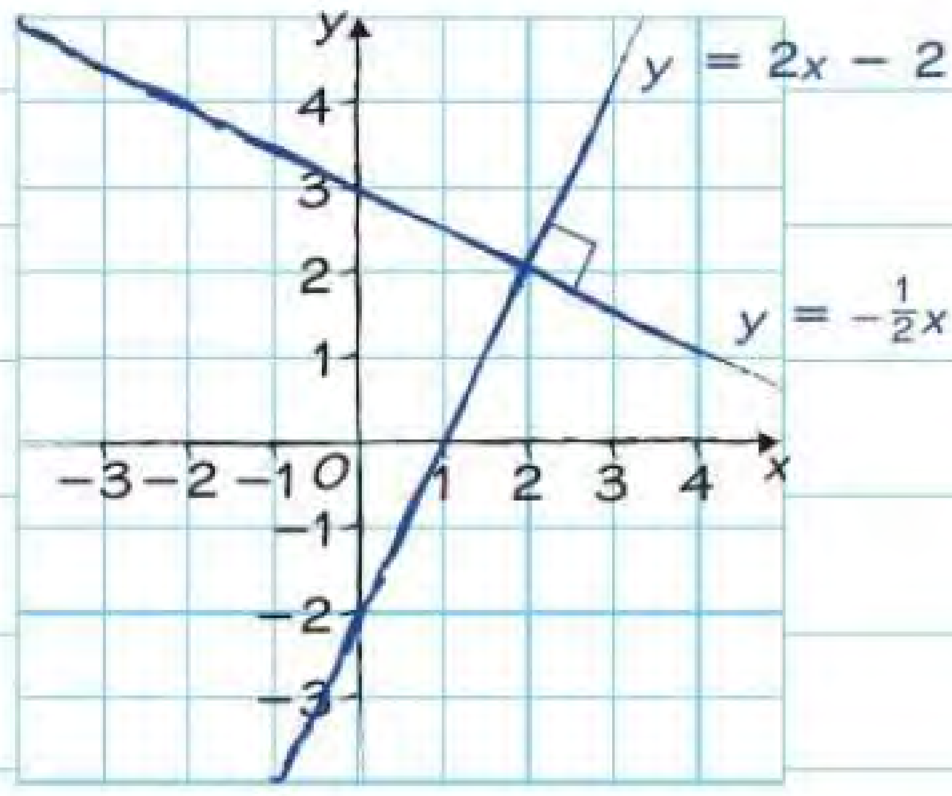
To find the gradient, rearrange the equation into the form $y = mx + c$.

Parallel and perpendicular

Parallel lines have the same gradient.
These three lines all have a gradient of 1.



Perpendicular means at right angles. If a line has gradient m then any line perpendicular to it will have gradient $-\frac{1}{m}$



Worked example

P is the point (3, −2) and Q is the point (7, 6). The line L is perpendicular to PQ and passes through the midpoint of PQ. Find an equation for L in the form $ax + by + c = 0$, where a, b and c are integers. **(5 marks)**

Gradient of $PQ = \frac{6 - (-2)}{7 - 3} = \frac{8}{4} = 2$

So gradient of $L = -\frac{1}{2}$

Midpoint of $PQ = \left(\frac{3 + 7}{2}, \frac{-2 + 6}{2}\right)$

$= (5, 2)$

Equation of L: $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x - 5)$

$2y - 4 = -x + 5$

$x + 2y - 9 = 0$

Find the gradient and the midpoint of PQ. The midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Worked example

The line L_1 has equation $7y + 2x - 3 = 0$. The line L_2 is perpendicular to L_1 and crosses the y-axis at (0, 5). Find an equation for L_2 in the form $ax + by + c = 0$, where a, b and c are integers. **(3 marks)**

L_1: $7y = -2x + 3$

$y = -\frac{2}{7}x + \frac{3}{7}$

L_2 has gradient $\frac{7}{2}$

L_2: $y = \frac{7}{2}x + 5$

$2y = 7x + 10$

$7x - 2y + 10 = 0$

Start by finding the gradient of L_1. The easiest way to do this is to rearrange the equation into the form $y = mx + c$.

Now try this

1 The line L has equation $y = 10 - 3x$

(a) Show that the point P (4, −2) lies on L. Answer **(1 mark)**

(b) Find an equation of the line perpendicular to L which passes through P. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. Answer **(3 marks)**

2 The line L_1 with equation $4x - 5y - 1 = 0$ crosses the x-axis at A. The line L_2 is perpendicular to L_2 and passes through A. Find the equation of L_2 in the form $y = mx + c$ **(4 marks)**

Answer

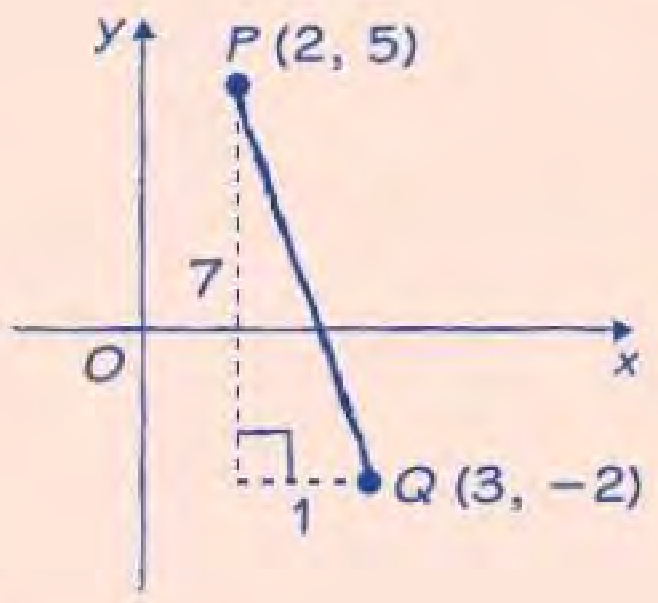
Substitute $y = 0$ into the equation of L_1 to work out the x-coordinate of A.

Lengths and areas

You might have to calculate the length of a line segment, or the area of a shape on a coordinate grid. It's always a good idea to **draw sketches** to keep track of your working.

Worked example

P is the point (2, 5) and Q is the point (3, −2). The length of PQ is $a\sqrt{2}$, where a is an integer. Find the value of a. **(3 marks)**



$PQ^2 = 1^2 + 7^2$

$= 50$

$PQ = \sqrt{50}$

$= \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

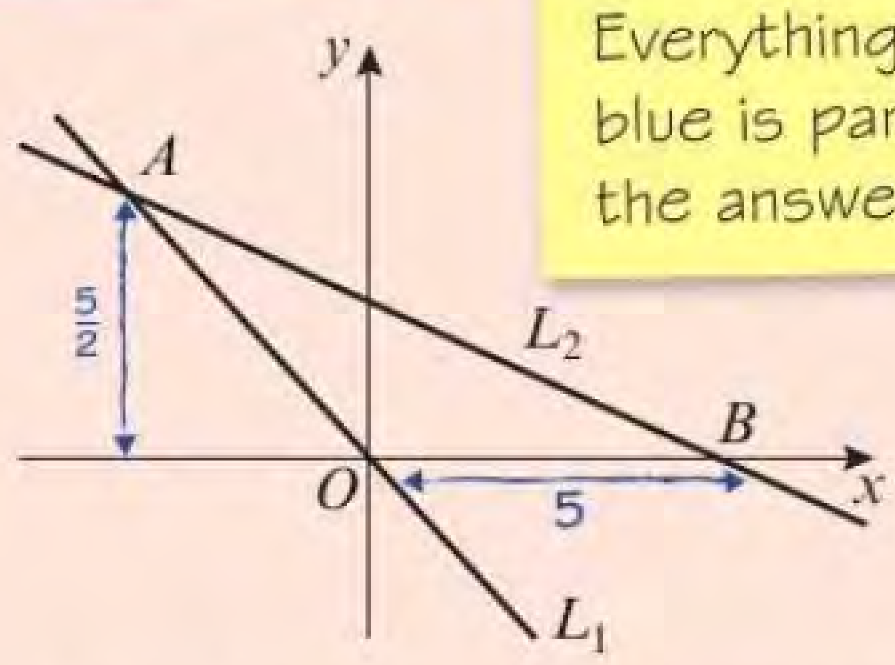
$a = 5$

Using a formula

If P has coordinates (x_1, y_1) and Q has coordinates (x_2, y_2), then the length of the line segment PQ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Worked example



Everything in blue is part of the answer.

The line L_1 has equation $y = -x$, and the line L_2 has equation $3y + x - 5 = 0$

(a) Work out the coordinates of A. **(2 marks)**

$3(-x) + x - 5 = 0$

$-2x - 5 = 0$

$x = -\frac{5}{2},\ y = \frac{5}{2}$

A is the point $\left(-\frac{5}{2}, \frac{5}{2}\right)$

(b) Find the area of triangle AOB. **(3 marks)**

$3(0) + x - 5 = 0$

$x = 5$

B is the point (5, 0).

Area $= \frac{1}{2} \times 5 \times \frac{5}{2} = \frac{25}{4}$

(a) To work out the coordinates of the point where two lines intersect you need to solve their equations **simultaneously**. Substitute $y = -x$ into the equation for L_2.

(b) The vertical height of triangle AOB is $\frac{5}{2}$. Substitute $y = 0$ into the equation for L_2 to find the coordinates of B, then use Area $= \frac{1}{2} \times$ base $\times$ height. You don't need to give any units when you're calculating lengths and areas on a graph.

Now try this

The line L_1 has equation $x - 2y + 6 = 0$

L_1 crosses the x-axis at P and the y-axis at Q.

(a) Show that $PQ = 3\sqrt{5}$ Answer **(3 marks)**

The line L_2 is perpendicular to L_1 and passes through Q.

(b) Find an equation for L_2. Answer **(4 marks)**

L_2 crosses the x-axis at R.

(c) Find the area of the triangle PQR. **(4 marks)**

Answer

Draw a sketch to help you visualise the problem.