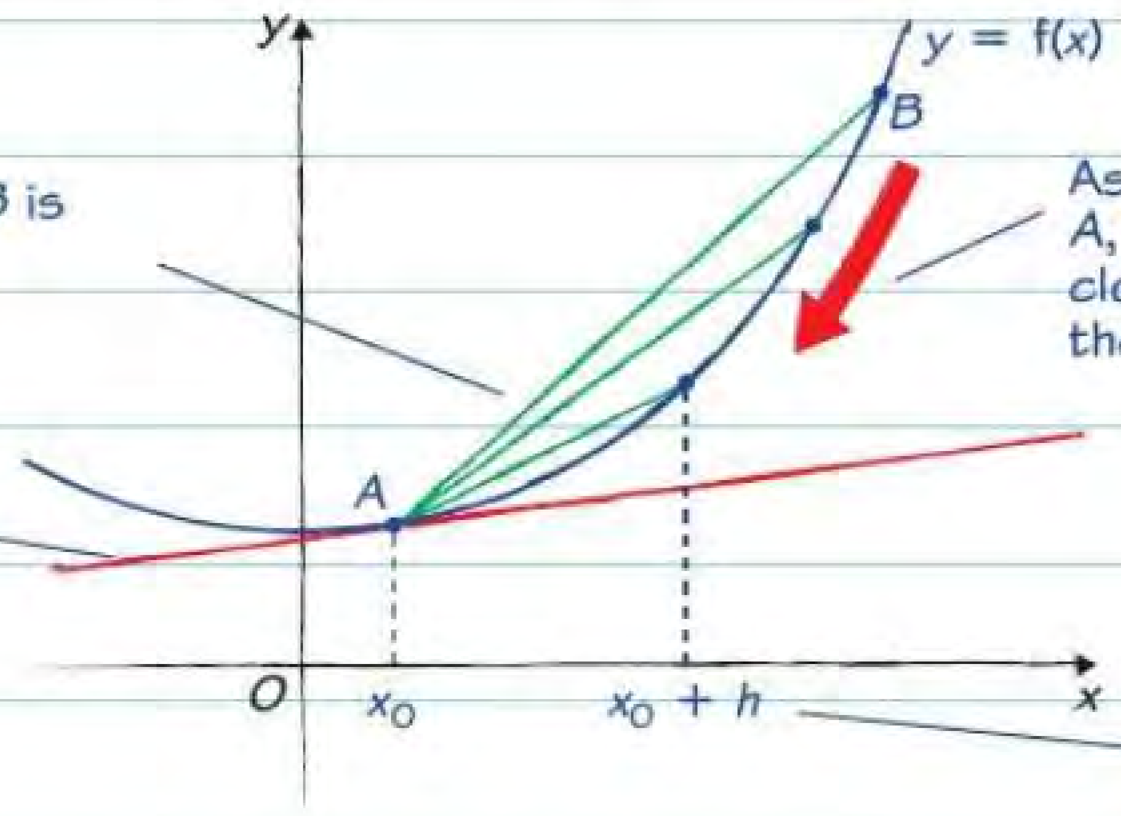
Differentiating from first principles

You can find the **gradient** of a curve at a point A by considering the gradient of the chord AB as B gets closer to A.



Worked example

Prove from first principles that the derivative of $4x^3$ is $12x^2$. **(4 marks)**

$$f(x) = 4x^3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4(x + h)^3 - 4x^3}{h}$$

$$= \lim_{h \to 0} \frac{4(x^3 + 3x^2h + 3xh^2 + h^3) - 4x^3}{h}$$

$$= \lim_{h \to 0} \frac{12x^2h + 12xh^2 + 4h^3}{h}$$

$$= \lim_{h \to 0} (12x^2 + 12xh + 4h^2)$$

As $h \to 0$, then $12xh \to 0$ and $4h^2 \to 0$.

So $f'(x) = 12x^2$, as required.

First principles formula

You can solve first principles problems using this formula, which is found in the formulae booklet.

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Problem solved!

Substitute into the first principles formula. If $f(x) = 4x^3$ then $f(x + h) = 4(x + h)^3$. Multiply out the brackets and simplify the fraction. Any terms with a positive power of h will tend towards 0 as h tends towards 0.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

1 Given that $f(x) = x^2$, use differentiation from first principles to find $f'(7)$. **(5 marks)**

Answer

The question states "first principles" so you need to use the formula, or write an expression for the chord AB. Make sure you write out which terms in the expression $\to 0$ as $h \to 0$.

2 The points A and B with x-coordinates 5 and $5 + h$ respectively lie on the curve with equation $y = 2x^2 + 3x$.

(a) Show that the gradient of AB is $23 + 2h$. **(3 marks)** Answer

(b) Deduce the gradient of the tangent to the curve at A. **(1 mark)** Answer