Summary of key points

1 **Bivariate data** is data which has pairs of values for two variables.

2 **Correlation** describes the nature of the linear relationship between two variables.

3 When two variables are correlated, you need to consider the context of the question and use your common sense to determine whether they have a causal relationship.

4 The **regression line** of y on x is written in the form $y = a + bx$.

5 The coefficient b tells you the change in y for each unit change in x.

- If the data is positively correlated, b will be positive.
- If the data is negatively correlated, b will be negative.

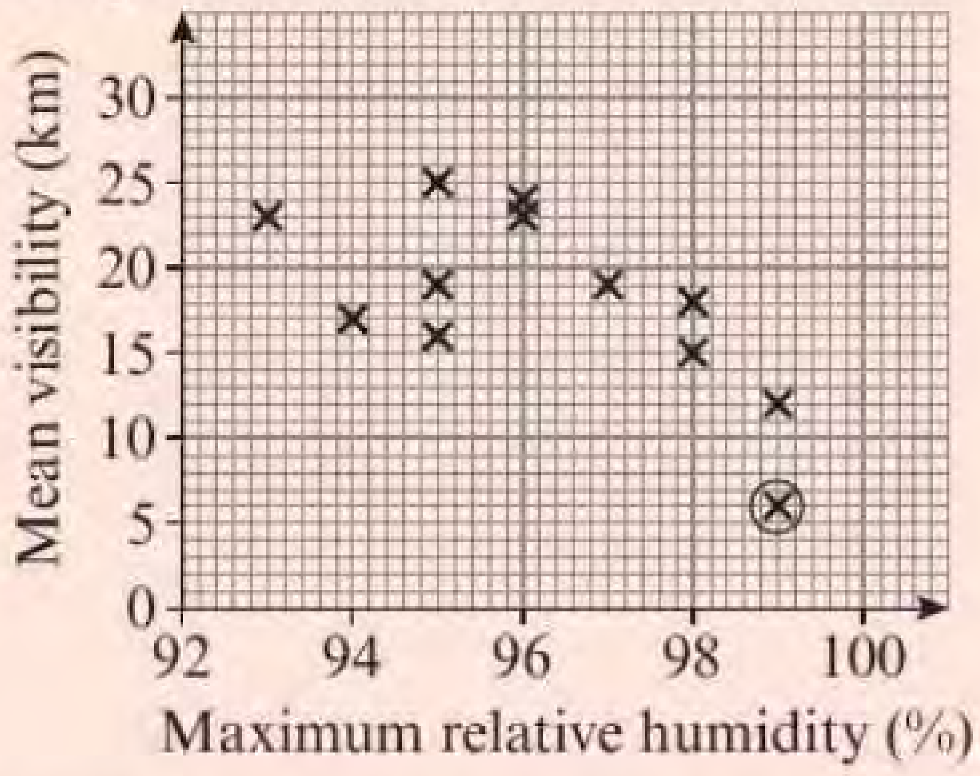
6 You should only use the regression line to make predictions for values of the dependent variable that are within the range of the given data.

Correlation and cleaning data

You can use correlation to describe a **linear relationship** between two variables. The closer the data approximates to a linear relationship, the **stronger** the correlation.

Worked example

The scatter diagram shows the maximum relative humidity, and mean visibility for the first 12 days in July, 2015 in Camborne.



(a) Describe and interpret any correlation shown on the scatter diagram. **(2 marks)**

Quite strong negative correlation. As humidity increases, visibility decreases.

(b) From your knowledge of the large data set, explain whether the relationship between humidity and visibility might be causal. **(2 marks)**

High humidity can give rise to misty and foggy conditions, which can reduce visibility, so the relationship could be causal.

The visibility for the circled data point is found to be an outlier.

(c) Justify the inclusion of this outlier when analysing the data. **(1 mark)**

A visibility of 6 km is not unlikely, and follows the trend of the data.

Cause and effect

Watch out! Correlation doesn't always mean that two variables are related.

Bottled water doesn't cause bee stings but, when the weather is hotter, bottled water sales and bee stings both increase. This is an example of a non-causal relationship.

You need to understand what the different variables in the large data set represent, and how they might be related.

There is more on the large data set on page 60.

Problem solved!

Removing obviously incorrect data values is sometimes called **cleaning data**. You might want to **keep** an outlier if:

- it appears to be genuine
- the data is from a reliable source, or has already been cleaned.

You might want to **remove** an outlier if:

- it is obviously a mistake
- you want to intentionally omit extreme data values

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

Michelle is training a group of young people to use image-editing software. She records the time, y hours, it takes each person in the group to reach a certain standard of proficiency.

Group member	A	B	C	D	E	F	G	H	I	J
Age (x years)	15	19	81	16	20	17	19	21	18	15
Time (y hours)	14	13	10	12	7	18	9	6	9	11

Michelle determines that $x = 81$ is an outlier, and decides to omit the data for group member C from her results. Answer

(a) Comment on the validity of Michelle's decision to omit the outlier. **(2 marks)**

(b) Draw a scatter diagram for the remaining 9 results. Answer **(3 marks)**

(c) Describe the correlation shown on your scatter diagram, and interpret this in the context of the model. Answer **(2 marks)**

Regression

A **regression line** on a scatter diagram is a type of line of best fit. It can be used as a **linear model** for the relationship between the two variables.

You don't need to be able to calculate regression lines in your exam, but you do need to be able to **interpret** their equations. The regression line (or **least squares regression line**) of y on x is written as:

$$y = a + bx$$

The coefficient b tells you the **gradient** of the regression line – this is how much the variable y increases for each unit increase in x.

Dependent and independent

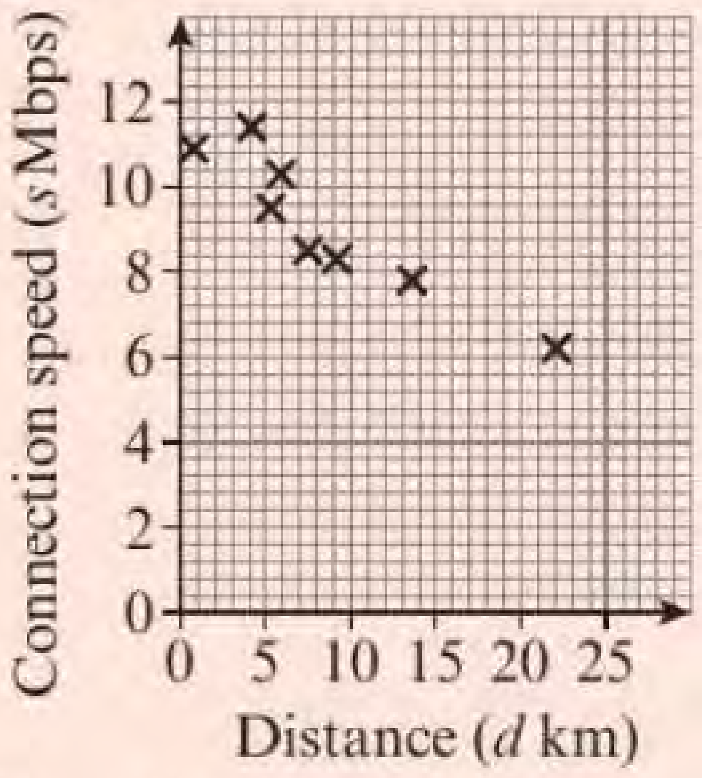
The order of the variables in a regression equation is important.

In the regression model $y = a + bx$, x is the **independent** (or **explanatory**) variable, and y is the **dependent** (or **response**) variable. In an experiment, x would be the variable you changed, and y would be the variable you recorded. You always plot the explanatory variable on the **horizontal** axis.

If the value of b (the gradient) in the regression equation is **negative**, then the regression line slopes **downwards**. This means that as one variable increases, the other decreases.

Worked example

An internet provider advertises an internet connection speed of 12 Mbps. Dhevan records the speed of 8 internet connections, s Mbps, at different distances, d km from the phone exchange. He records his results on a scatter diagram:



The equation of the regression line of s on d for these data is $s = 11.2 - 0.242d$

(a) Give an interpretation of the gradient of the regression line. **(1 mark)**

According to this model, for each additional km from the phone exchange, your connection speed will reduce by 0.242 Mbps.

(b) Explain why a regression model of the form $s = a + bd$ is supported for these data. **(1 mark)**

The data show strong negative correlation. Hence a linear model is suitable.

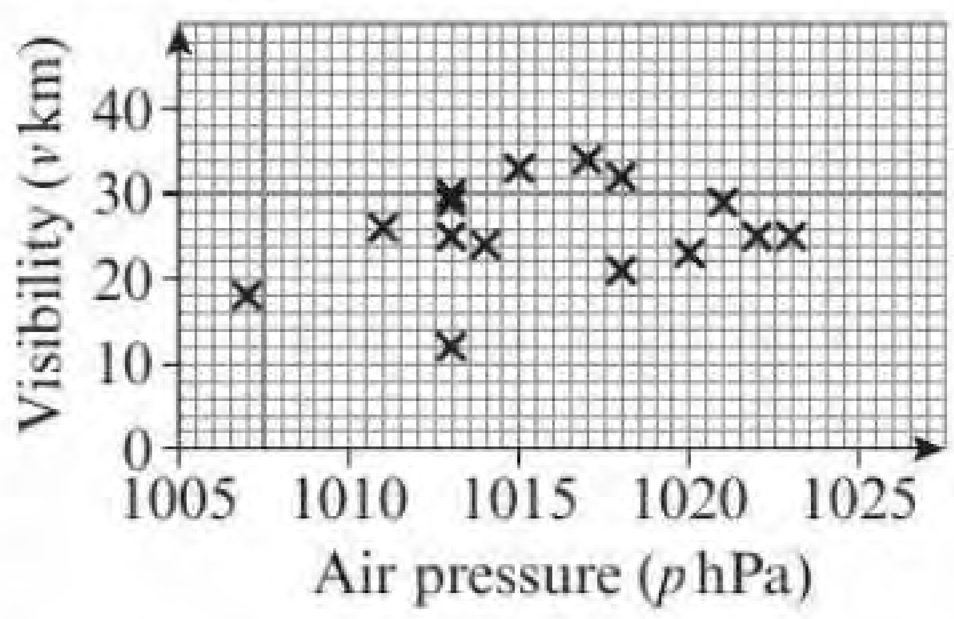
You could also consider whether the relationship is likely to be causal. Distance could affect connection speed, so this supports a relationship between the variables.

Validity

The **strength of the correlation** between two variables tells you how accurately a **linear** model represents the relationship between them. The stronger the correlation, the more closely the regression line models the data.

Now try this

The scatter diagram shows the daily mean visibility, v km, and daily mean air pressure p hPa, in Hurn for the first 15 days of August 2015.



The equation of the regression line of v on p for these data is $v = -353.5 + 0.373p$

(a) Interpret the value 0.373 in this model. Answer **(1 mark)**

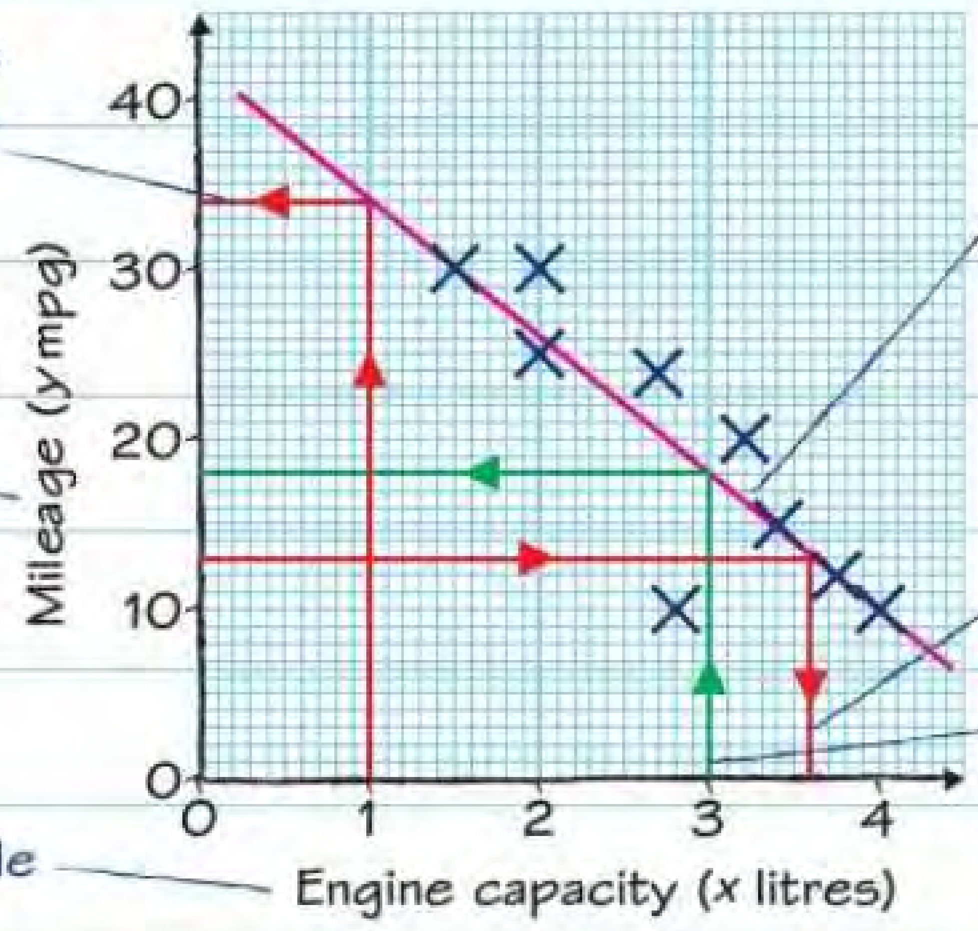
(b) Comment on the validity of a linear regression model for these data. **(1 mark)**

Answer

Had a look ☐ Nearly there ☐ Nailed it! ☐

Using regression lines

If you know the value of the **independent** variable, you can use a regression line (or its equation) to **estimate** or **predict** the corresponding value of the dependent variable. You won't need to make predictions like this in your exam, but you do need to know when they are **reliable**. This scatter diagram shows the mileage in miles per gallon (mpg) of 9 cars, and their engine capacities.



Worked example

The age, x years, and shell size, y mm, of 8 Dungeness crabs were recorded.

y	151.8	150.4	140.3	133.4	155.9	153.3	141.6	127.7
x	3.3	3.0	2.4	2.3	3.3	3.0	2.7	2.2

The equation of the regression line of y on x for these data is $y = 83.17 + 22.03x$.

(a) Give an interpretation of the gradient of the regression line. **(1 mark)**

A typical crab shell will grow about 22 mm per year.

(b) Comment on the reliability of using this regression equation to estimate the shell size of a Dungeness crab with age:

(i) 1.6 years (ii) 2.5 years **(2 marks)**

(i) 1.6 years is outside the range of the data, so this is an example of extrapolation. The estimate will be unreliable.

(ii) 2.5 years is inside the range of the data, so this is an example of interpolation. The estimate will be reliable.

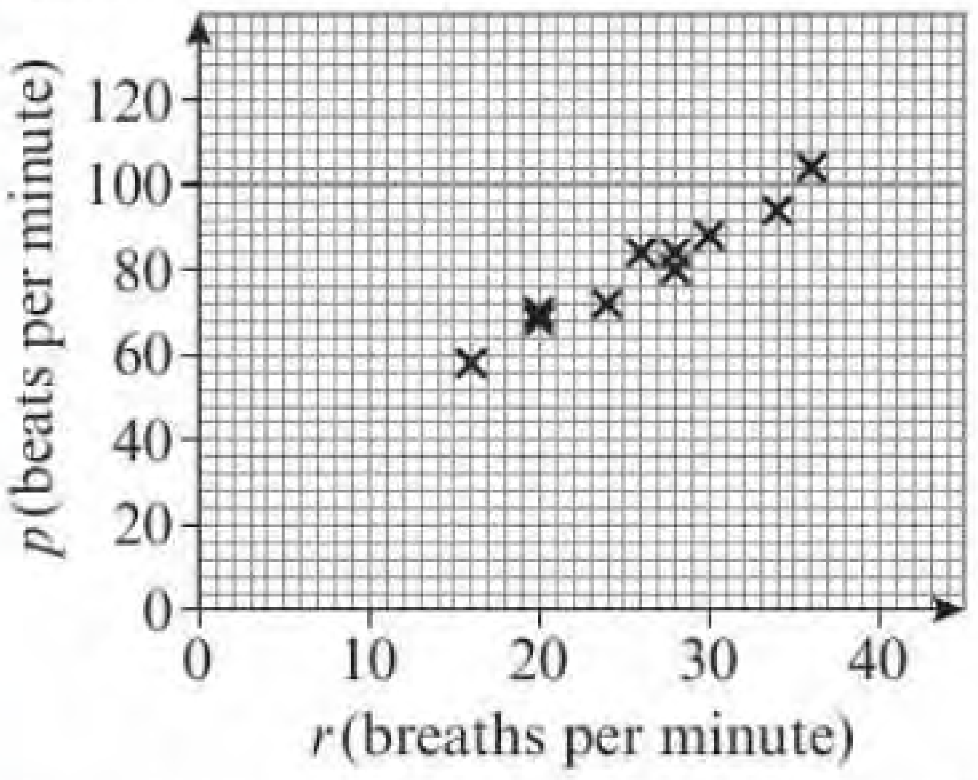
(d) Give a reason why this regression model would not be suitable for estimating the age of a Dungeness crab with a shell size of 145 mm. **(1 mark)**

Age is the independent variable in this model. You should only use a regression model to estimate values of the dependent variable.

You would need to use the regression line of **x on y** to estimate age given shell size.

Now try this

In a study, participants exercise for 5 minutes then record their breath rate (r breaths per minute) and pulse (p beats per minute). The data for 10 participants are shown in the scatter diagram.



The equation of the regression line of p on r for these data is $p = 25.5 + 2.09r$

(a) Give a reason why a linear regression model is likely to be suitable for these data. Answer **(1 mark)**

(b) The equation of the regression line is used to estimate the pulse rate of a participant making

(i) 22 breaths per minute Answer

(ii) 10 breaths per minute.

Comment on the reliability of these estimates. Answer **(2 marks)**

Summary of key points

1 If $y = ax^n$ for constants a and n then $\log y = \log a + n \log x$

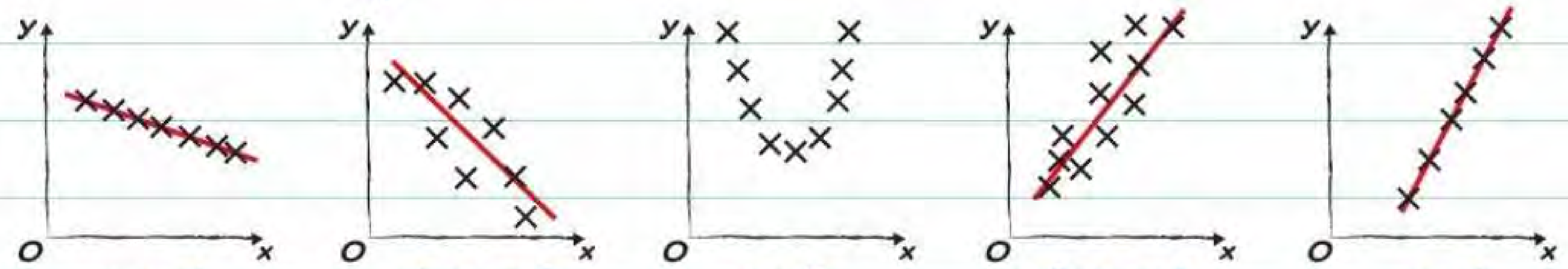
2 If $y = kb^x$ for constants k and b then $\log y = \log k + x \log b$

3 The **product moment correlation coefficient** describes the linear correlation between two variables. It can take values between –1 and 1.

Measuring correlation

You can use the **product moment correlation coefficient (PMCC)** to measure the strength of the linear correlation between two variables. You need to be able to use your calculator to find the PMCC.

Negative correlation — No linear correlation — Positive correlation



$r = -1$ — Perfect negative correlation | $-1 < r < 0$ | $r = 0$ | $0 < r < 1$ | $r = 1$ — Perfect positive correlation

The closer r is to -1, the **stronger** the negative correlation.

$r = 0$ only implies no **linear** correlation. There might be a **non-linear** relationship between the variables, as there is here.

The closer r is to 0, the **weaker** the positive correlation.

Worked example

An engineer is using a wind tunnel to investigate the effects of air speed on drag over an aircraft wing. She records her results in a table.

Air speed, a (mph)	20	40	80	120	160
Drag, d (N)	80	260	1670	5400	12700

The engineer believes the data can be modelled by a relationship of the form $d = Pa^k$ for constants P and k. She codes the data using $x = \log a$ and $y = \log d$.

(a) Calculate the product moment correlation coefficient of the coded data. **(3 marks)**

x	1.301	1.602	1.903	2.079	2.204
y	1.903	2.415	3.223	3.732	4.104

$r = 0.994$

The equation of the regression line of y on x is found to be $y = 2.470x - 1.415$

(b) Estimate the values of P and k in the engineer's model, and comment on the validity of this model. **(4 marks)**

$\log d = 2.470 \log a - 1.415$

$\log d = \log a^{2.470} - \log 10^{1.415}$

$\log d = \log \dfrac{a^{2.470}}{10^{1.145}}$

$d = 10^{-1.415}\, a^{2.470}$

So $P = 10^{-1.415} = 0.0385$ and $k = 2.470$

The PMCC is close to 1 ($r = 0.994$), which suggests this model is valid.

Use your calculator to find the product moment correlation coefficient. You might have to select the type of model you are using. Because the PMCC measures **linear** correlation, you should choose a model of the form $y = a + bx$. The letter r is usually used to denote the PMCC.

Non-linear models

You can use **logarithms** to test for non-linear relationships between two variables x and y. You need to know about two different forms:

1. If $y = ax^n$ for constants a and n then $\log y = \log a + n \log x$

You need to use the coding $Y = \log y$ and $X = \log x$ to obtain a linear relationship.

2. If $y = kb^x$ for constants k and b then $\log y = \log k + x \log b$

You need to use the coding $Y = \log y$ and $X = x$ to obtain a linear relationship.

You can revise the techniques needed to determine these relationships on page 53.

Now try this

The daily maximum relative humidity (%) and the daily mean visibility (km) recorded in Leeming for the first 8 days in May 2015 were recorded in the large data set.

Humidity (%)	99	94	95	94	99	87	88
Visibility (km)	25	16	14	20	15	24	32

Calculate the product moment correlation coefficient of the data, and describe the nature of the linear relationship (if any).

Answer **(2 marks)**