Summary of key points

A single fraction with two distinct linear factors in the denominator can be split into two 6 separate fractions with linear denominators. This is called splitting it into partial fractions:

$$\frac{5}{(x+1)(x-4)} = \frac{A}{x+1} + \frac{B}{x-4}$$

The method of partial fractions can also be used when there are more than two distinct linear 7 factors in the denominator:

$$\frac{7}{(x-2)(x+6)(x+3)} = \frac{A}{x-2} + \frac{B}{x+6} + \frac{C}{x+3}$$

A single fraction with a repeated linear factor in the denominator can be split into two or 8 more separate fractions:

$$\frac{2x+9}{(x-5)(x+3)^2} = \frac{A}{x-5} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

An improper algebraic fraction is one whose numerator has a degree equal to or larger than 9 the denominator. An improper fraction must be converted to a mixed fraction before you can express it in partial fractions.

10 You can either use:

- algebraic division
- or the relationship $F(x) = Q(x) \times \text{divisor} + \text{remainder}$ to convert an improper fraction into a mixed fraction.

Nearly there

Nailed it!





Many algebraic fractions can be written as the sum of simpler fractions. This technique is called writing a fraction in partial fractions. In your A-level exam you might have to use partial fractions with the binomial expansion or when integrating. You can revise these topics on pages 75 and 105.

Worked example

Had a look

$f(x) = \frac{7 - 2x}{(2x - 1)(x + 1)}$	
Express $f(x)$ in partial fractions.	(3 marks)
$\frac{7-2x}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$	
(2x-1)(x+1) = 2x-1 + 1 7 - 2x = A(x + 1) + B(2x	- 1)
Let $x = \frac{1}{2}$: $7 - 2(\frac{1}{2}) = A(\frac{1}{2} + 1)$ <u>$A = 4$</u>	
Let $x = -1$: 7 - 2(-1) = B(2(-1))	1) - 1)
$f(x) = \frac{4}{2x - 1} - \frac{3}{x + 1}$ $\frac{B = -3}{x + 1}$	

Golden rule

Find as many missing values as possible by substituting values for x to make some of the factors equal to zero. The more factors you can find this way, the easier it will be to equate coefficients later.

The denominators on the right-hand side are factors of the original denominator. If all the factors are different, then each one appears as a denominator once.

Cover up and calculate

If the expression has no repeated factors, you can use this quick method to find numerators. Choose a factor, and work out the value of xwhich makes that factor equal to zero. Then cover it up with your finger, and evaluate what's left of the fraction with that value of x.

 $\frac{7-2(-1)}{2(-1)-1} = \frac{9}{-3} = -3$ $f(x) = \frac{7 - 2x}{(2x - 1)(0)}$

Covering up (x + 1) in the Worked example above and evaluating what's left with x = -1 gives you B.

Problem solved!

You need to do a bit more work if there is a repeated factor, like $(x - 3)^2$, or if the fraction is improper. You should always work out any values you can by substituting first. Here you can work out one value by substituting x = 3. To work out the other values you need to equate coefficients. You could multiply out both sides first:

 $2x^2 - 1 = Ax^2 + (-6A + B)x + (9A - 3B + C)$

You will need to use problem-solving skills throughout your exam - be prepared!

Now try this



Worked example

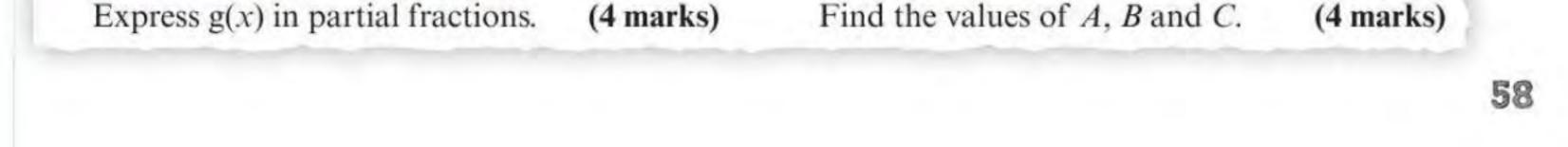
 $\frac{2x^2 - 1}{(x - 3)^2} = A + \frac{B}{(x - 3)} + \frac{C}{(x - 3)^2}, \quad x \neq 3$ Find the values of A, B and C. (4 marks) $2x^2 - 1 = A(x - 3)^2 + B(x - 3) + C$ Let x = 3: $2(3)^2 - 1 = C$ C = 17Equate x^2 terms: A = 2Equate constant terms: -1 = 9A - 3B + C-1 = 18 - 3B + 17-36 = -3BB = 12

There is a repeated factor, so you need one fraction with denominator (x + 2) and another fraction with denominator $(x + 2)^2$. $1 g(x) = \frac{8x^2}{(3x-2)(x+2)^2}$

$$2 \frac{6x^2 - 1}{(2x - 3)(x + 1)} = A + \frac{B}{2x - 3} + \frac{C}{x + 1}$$

Find the values of A, B and C.

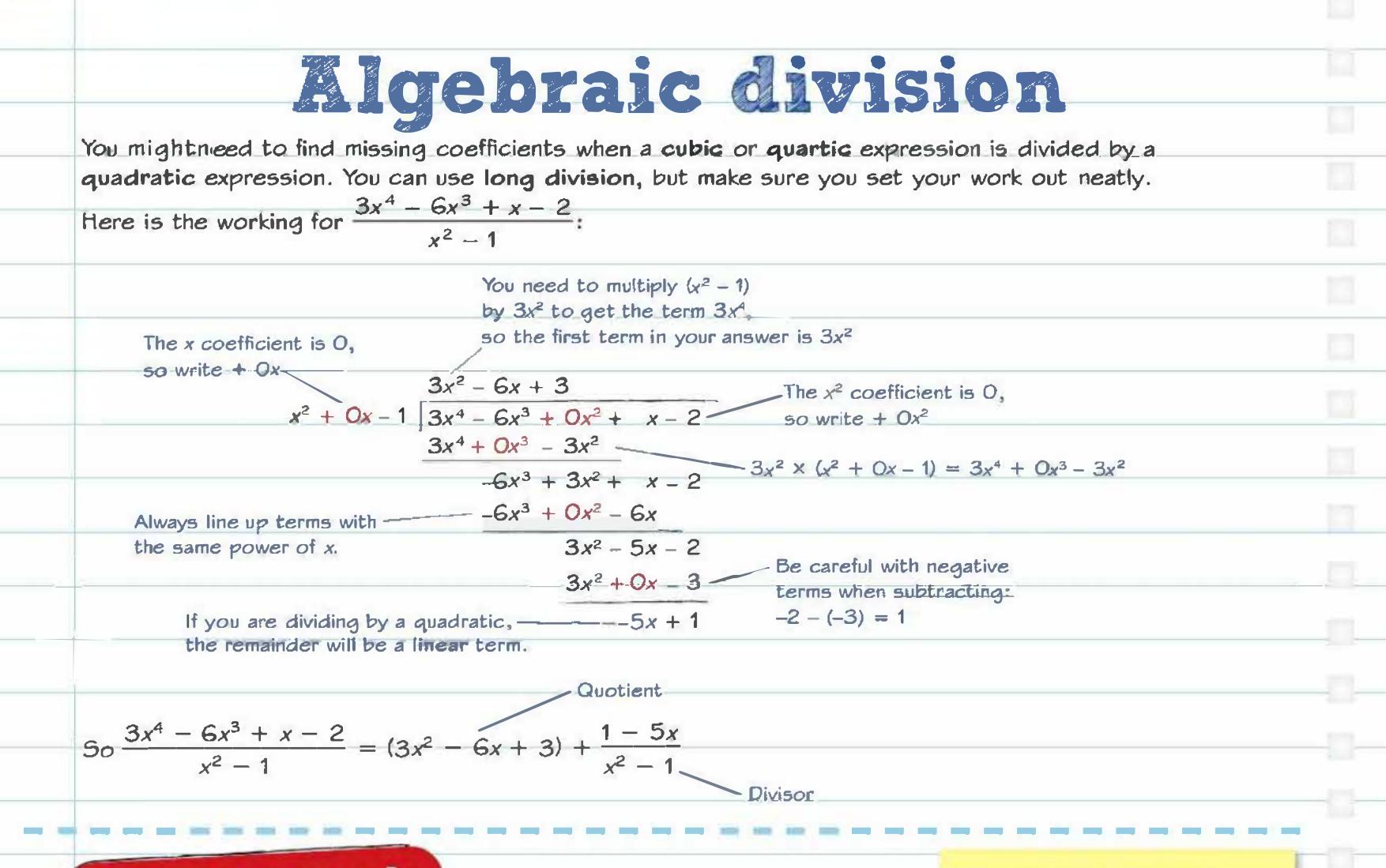
(4 marks)



Pure

Nearly there





Worked example

missing coefficients. Follow Given that these steps: $\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$ 1. Multiply both sides by the divisor. find the values of the constants a, b, c, d and e. (4 marks) 2. Expand the brackets $3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$ carefully then collect $\equiv ax^4 + bx^3 + cx^2 - 4ax^2 - 4bx - 4c + dx + e$ like terms. $\equiv ax^4 + bx^3 + (c - 4a)x^2 + (d - 4b)x + (e - 4c)$ 3. Compare coefficients on x terms $\rightarrow d - 4b = 0$ x^4 terms $\rightarrow a = 3$ both sides. starting with the highest power of x. x^3 terms $\rightarrow b = -2$ d + 8 = 0 x^2 terms $\rightarrow c - 4a = -5$ d = -8As long as you write down c - 12 = -5 Constant terms $\rightarrow e - 4c = -4$ what each constant is e - 28 = -4c = 7equal to, you don't need e = 24to write out the whole

Now try this

Given that

$$\frac{2x^4 + 4x^2 - x + 2}{x^2 - 1} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 1}, \quad x \neq \pm 1$$

find the values of the constants a, b, c, d and e.

(4 marks)

Whichever method you use. make sure you either:

expression at the end.

You can also compare

coefficients to find the

- write out the expression in full with the constants in place. or
- write a = ..., b = ..., etc.

