

Summary of key points

- 9** To sketch the graph of $y = |f(x)|$
- Sketch the graph of $y = f(x)$
 - Reflect any parts where $f(x) < 0$ (parts below the x -axis) in the x -axis
 - Delete the parts below the x -axis
- 10** To sketch the graph of $y = f(|x|)$
- Sketch the graph of $y = f(x)$ for $x \geq 0$
 - Reflect this in the y -axis
- 11** $f(x + a)$ is a horizontal translation of $-a$.
- 12** $f(x) + a$ is a vertical translation of $+a$.
- 13** $f(ax)$ is a horizontal stretch of scale factor $\frac{1}{a}$
- 14** $af(x)$ is a vertical stretch of scale factor a .
- 15** $f(-x)$ reflects $f(x)$ in the y -axis.
- 16** $-f(x)$ reflects $f(x)$ in the x -axis.

Modulus transformations

You revised these transformations of the graph of $y = f(x)$ on pages 13 and 14:

- $y = f(x) + a$ Translation $\begin{pmatrix} 0 \\ a \end{pmatrix}$
- $y = f(x + a)$ Translation $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
- $y = af(x)$ Vertical stretch, scale factor a
- $y = f(ax)$ Horizontal stretch, scale factor $\frac{1}{a}$
- $y = -f(x)$ Reflection in the x -axis
- $y = f(-x)$ Reflection in the y -axis.

You need to be able to combine these transformations and use the modulus function to sketch more complicated transformations.

Golden rule

Carry out transformations in this order:

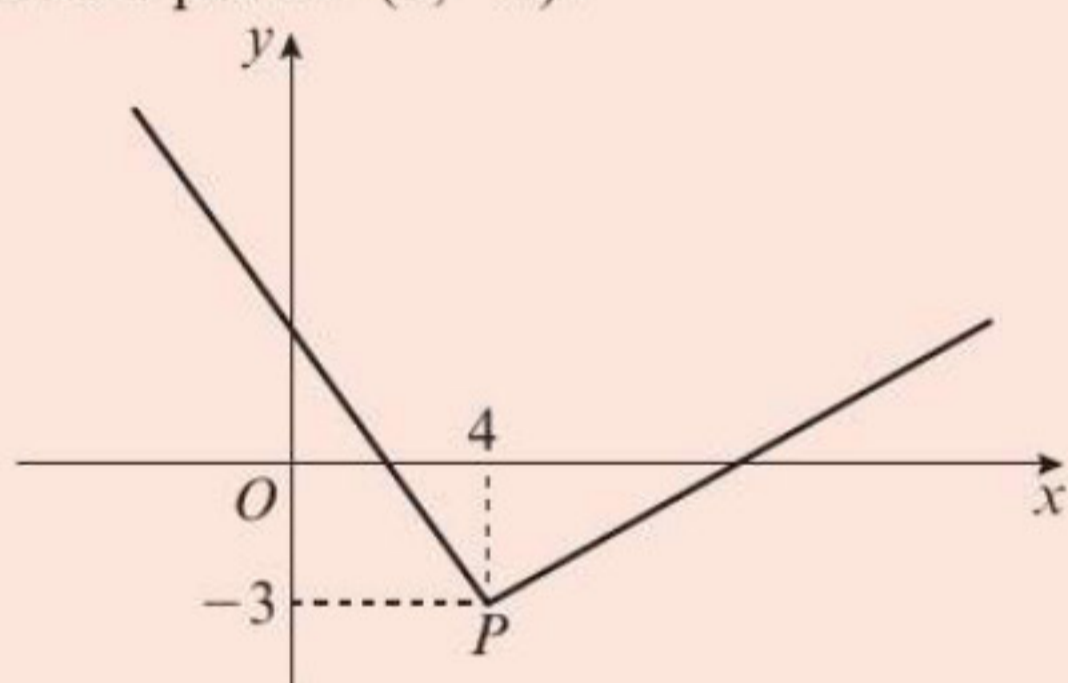
- 1** Anything 'inside' the function brackets
- 2** Multiples or modulus of the whole function
- 3** Addition or subtraction outside the function brackets.

$$y = \frac{1}{2}f(|x|) + 4$$

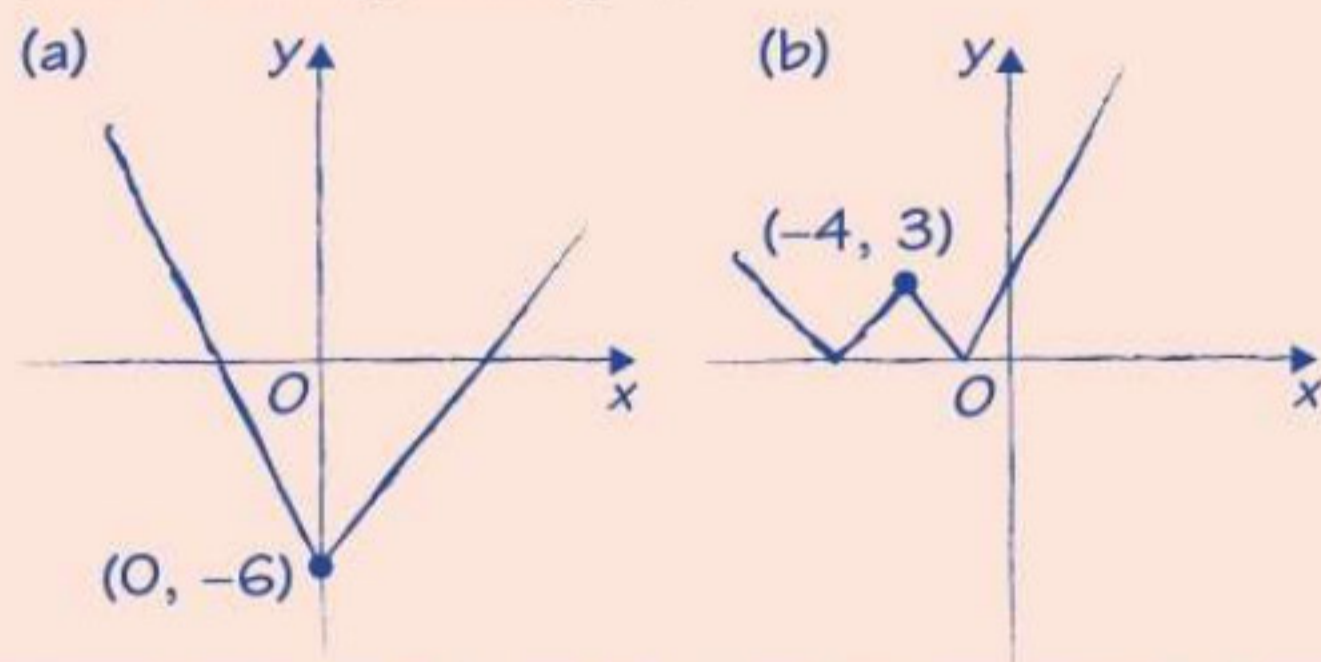
2 1 3

Worked example

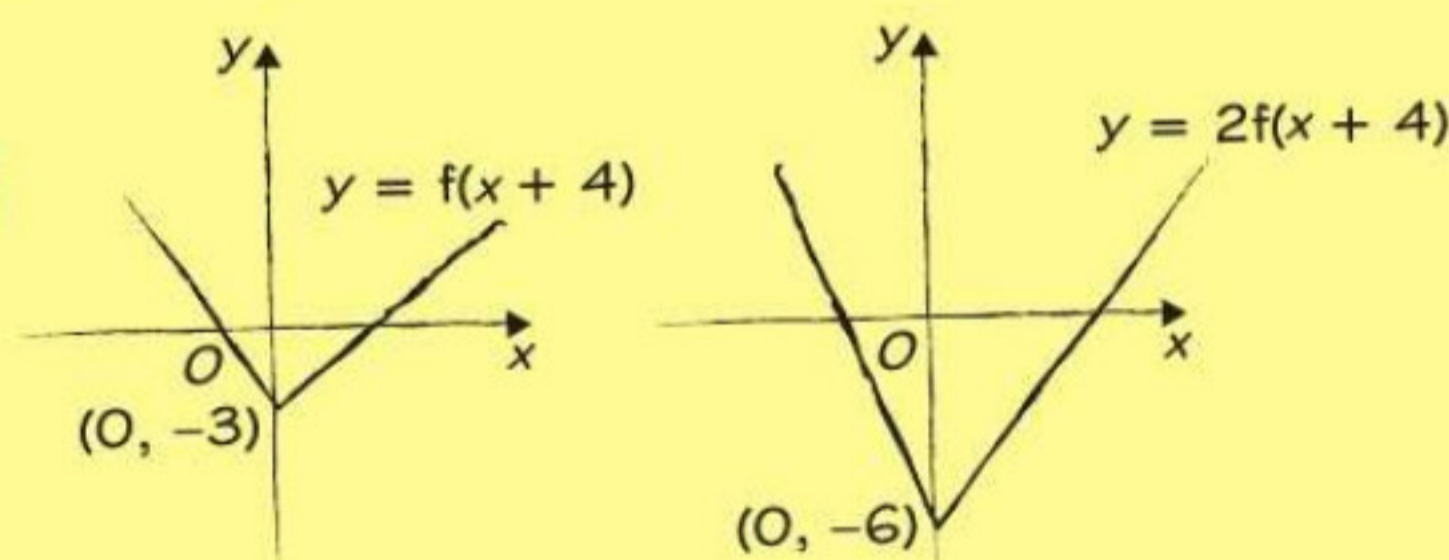
The diagram shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $P(4, -3)$.



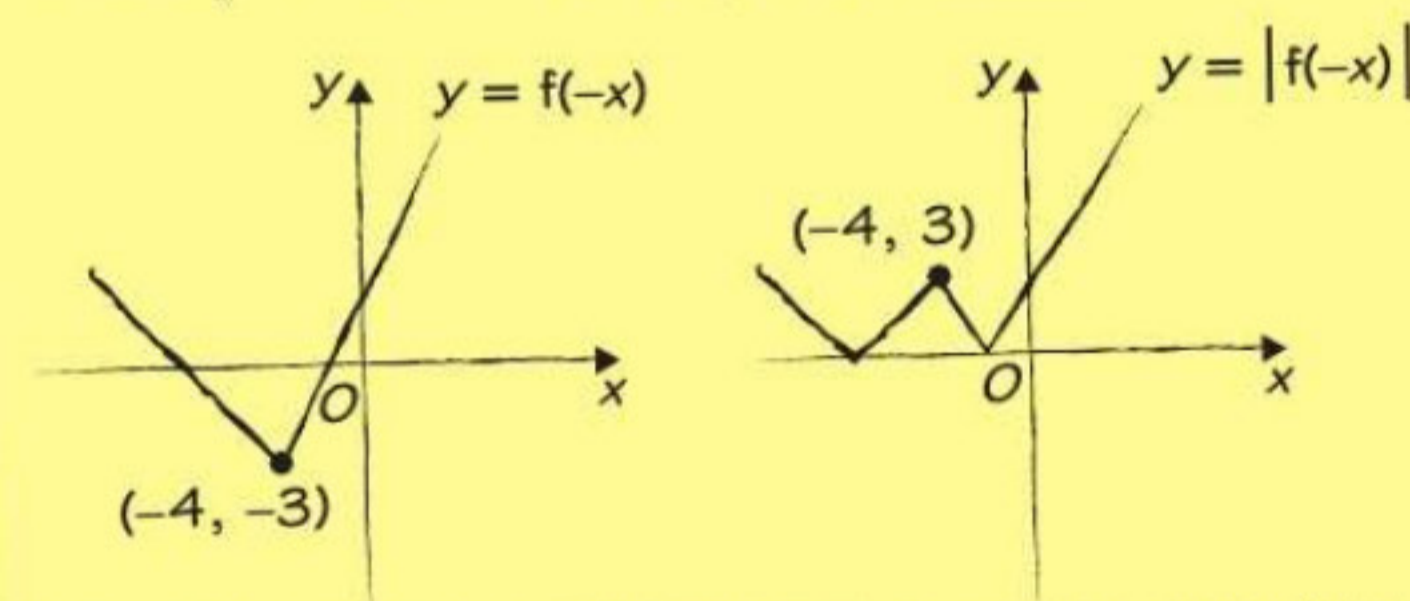
Sketch, on separate diagrams, the graphs of
 (a) $y = 2f(x + 4)$ (3 marks) (b) $y = |f(-x)|$ (3 marks)
 On each diagram, show the coordinates of the point corresponding to P .



For part (a) you need to carry out a translation $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ followed by a vertical stretch with scale factor 2.



For part (b) you need to carry out a reflection in the y -axis followed by a modulus.



Have a look at page 64 for a reminder about sketching the modulus of a function.

Now try this

The diagram shows a sketch of $y = f(x)$. The graph has turning points at P and Q .

- (a) Write down the coordinates of the point to which Q is transformed on the curve with equation
- (i) $y = 2f(2x)$ (ii) $y = |f(x + 4)|$ (4 marks)
- (b) Sketch, on separate diagrams, the graphs of
- (i) $y = f(-x) + 3$ (ii) $y = -|f(x)|$ (6 marks)

Indicate on each diagram the coordinates of any turning points.

