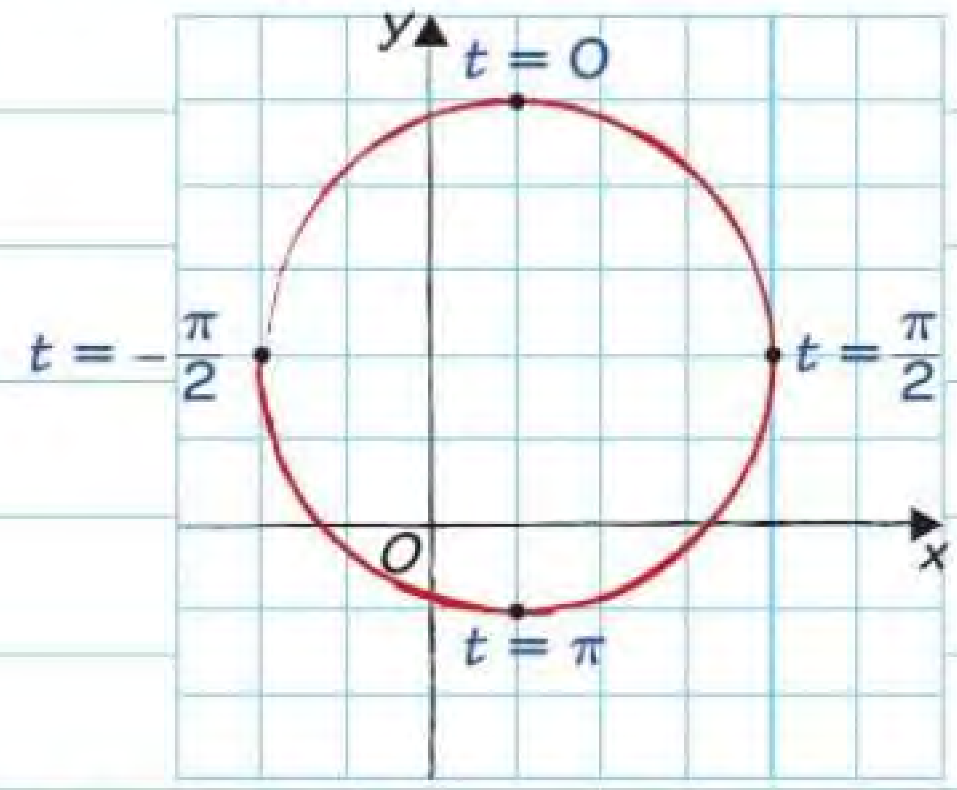
Parametric equations 1

You can define a curve by giving the x-coordinate and y-coordinate as separate functions of another variable, called a **parameter**.



Golden rules

1. Each value of t generates a point on the curve.
2. The value of t is **neither** the x- **nor** the y-coordinate of the curve.

The curve with parametric equations $y = 3\cos t + 2$, $x = 3\sin t + 1$, $-\pi < t \leqslant \pi$ is a circle, centre (1, 2) with radius 3.

Converting to cartesian

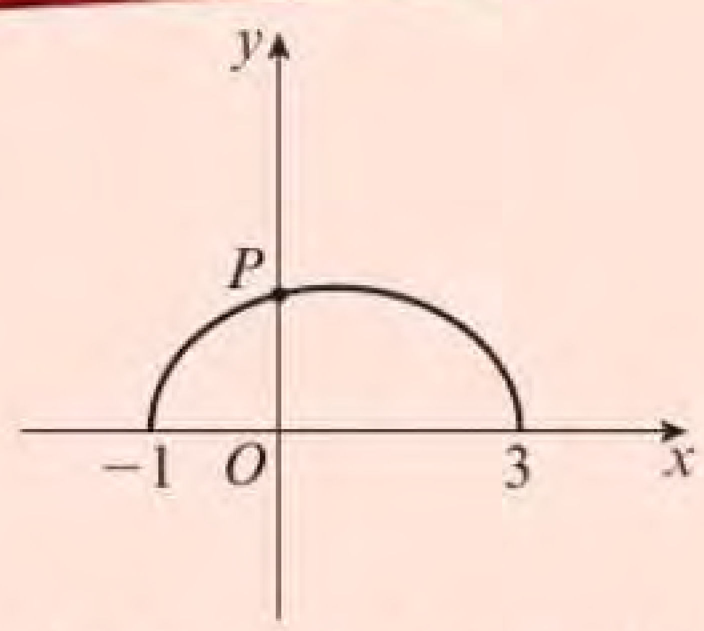
Cartesian equations only involve x and y. To convert to cartesian form you need to **eliminate** the parameter, t.

1. If the equations contain trig functions use an identity like $\sin^2 t + \cos^2 t \equiv 1$
2. Otherwise, write t in terms of either x or y then substitute into the other equation.

The curve crosses the y-axis when $x = 0$. Substitute $x = 0$ into the x-equation to find the value of t at this point. Then substitute this value of t into the y-equation to find the value of y at this point.

Find an expression for $\cos t$ in terms of x and an expression for $\sin t$ in terms of y. You can then use $\sin^2 t + \cos^2 t \equiv 1$ to eliminate t. Finally, rearrange the cartesian equation into the form asked for in the question.

Worked example



The curve C shown has parametric equations

$$x = 1 + 2\cos t, \quad y = \sin t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find the exact coordinates of the point P where the curve crosses the y-axis. **(3 marks)**

When $x = 0$, $1 + 2\cos t = 0$

$$\cos t = -\frac{1}{2}$$

$$t = \frac{2\pi}{3}$$

When $t = \frac{2\pi}{3}$, $y = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

P is $\left(0, \frac{\sqrt{3}}{2}\right)$

(b) Find a cartesian equation of C in the form $y^2 = f(x)$. **(3 marks)**

$\cos t = \frac{x-1}{2}$ $\quad \sin t = y$

$\cos^2 t + \sin^2 t = 1$

$$\left(\frac{x-1}{2}\right)^2 + y^2 = 1$$

$$y^2 = 1 - \left(\frac{x-1}{2}\right)^2$$

Worked example

The curve C has parametric equations

$$x = \ln t, \quad y = t^2 - 3$$

Find a cartesian equation of C. **(3 marks)**

$t = e^x$

$y = (e^x)^2 - 3 = e^{2x} - 3$

Start by using the addition formula for $\sin(A + B)$ to write y in terms of $\sin t$ and $\cos t$.

Now try this

$$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{3}\right), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

Show that a cartesian equation of the curve is

$$y = \frac{1}{2}x + \frac{\sqrt{3}}{2}\sqrt{1 - x^2}, \quad -1 < x < 1 \quad \textbf{(3 marks)}$$

Answer