Summary of key points

8 When θ is small and measured in radians:

- $\sin\theta \approx \theta$
- $\tan\theta \approx \theta$
- $\cos\theta \approx 1 - \frac{\theta^2}{2}$

Small angle approximations

The proof shown above makes use of small angle approximations for sin and cos. When x is small and measured in radians:

$\sin x \approx x \qquad \cos x \approx 1 - \frac{1}{2}x^2 \qquad \tan x \approx x$

You can use these approximations to deduce the limits given in the proof above. When h is small:

$$\frac{\sin h}{h} \approx \frac{h}{h} = 1 \text{ and}$$

$$\frac{\cos h - 1}{h} \approx \frac{1 - \frac{1}{2}h^2 - 1}{h} = -\frac{1}{2}h$$

This tends to 0 as $h \to 0$

Worked example

Show that, for small values of θ measured in radians, $\dfrac{1 - 4\cos\theta + \sin\theta}{3 + \tan 2\theta} \approx -1$ **(4 marks)**

$$\frac{1 - 4\cos\theta + \sin\theta}{3 + \tan 2\theta} \approx \frac{1 - 4(1 - \frac{1}{2}\theta^2) + \theta}{3 + 2\theta}$$

$$= \frac{2\theta^2 + \theta - 3}{3 + 2\theta}$$

$$= \frac{(2\theta + 3)(\theta - 1)}{2\theta + 3}$$

$$= \theta - 1$$

$\theta - 1 \to -1$ as $\theta \to 0$

If θ is small then you can assume that small multiples of θ such as 2θ and 3θ are also small.

Now try this