Summary of key points

11 If x and y are given as functions of a parameter, t: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Parametric differentiation

You can use this version of the **chain rule** to find the gradient of a curve defined by **parametric equations**.

This formula gives you the derivative **in terms of** t. You need to know the value of t for a particular point on the curve to find the gradient of the curve at that point.

Golden rule

$$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$$

Worked example

A curve C has parametric equations

$x = \sin^2 t,\ y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$

(a) Find $\frac{dy}{dx}$ in terms of t. **(4 marks)**

$\frac{dx}{dt} = 2\sin t\cos t,\ \frac{dy}{dt} = 2\sec^2 t$

$\frac{dy}{dx} = \frac{2\sec^2 t}{2\sin t\cos t} = \frac{1}{\sin t\cos^3 t}$

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P. **(6 marks)**

At the point where $t = \frac{\pi}{3}$:

$x = \sin^2\left(\frac{\pi}{3}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

$y = 2\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$

$\frac{dy}{dx} = \frac{1}{\sin\left(\frac{\pi}{3}\right)\cos^3\left(\frac{\pi}{3}\right)} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{8}\right)} = \frac{16}{\sqrt{3}}$

Equation of tangent:

$y - 2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$

At P, $y = 0$, so $-2\sqrt{3} = \frac{16}{\sqrt{3}}\left(x - \frac{3}{4}\right)$

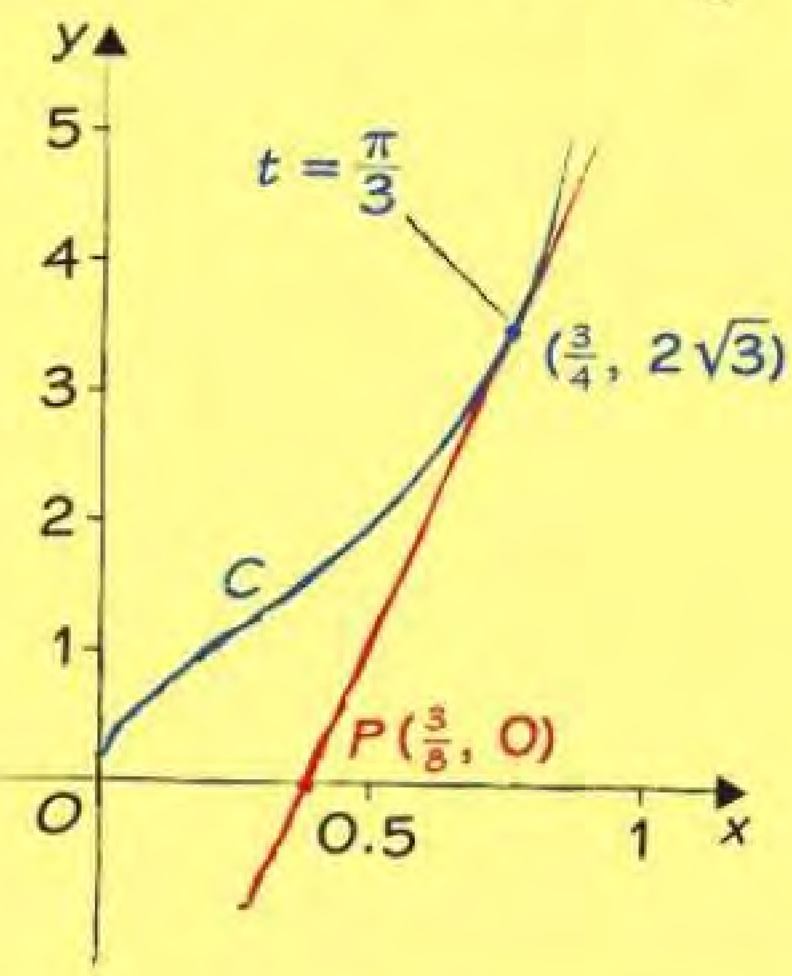
$-\frac{3}{8} = x - \frac{3}{4}$

$x = \frac{3}{8}$

If you have to differentiate a curve given in parametric form you will usually be asked to to find $\frac{dy}{dx}$ in terms of t. Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and write them down. Then work out $\frac{dy}{dt} \div \frac{dx}{dt}$.

You don't have to simplify your answer, but it might speed up your working later in the question.

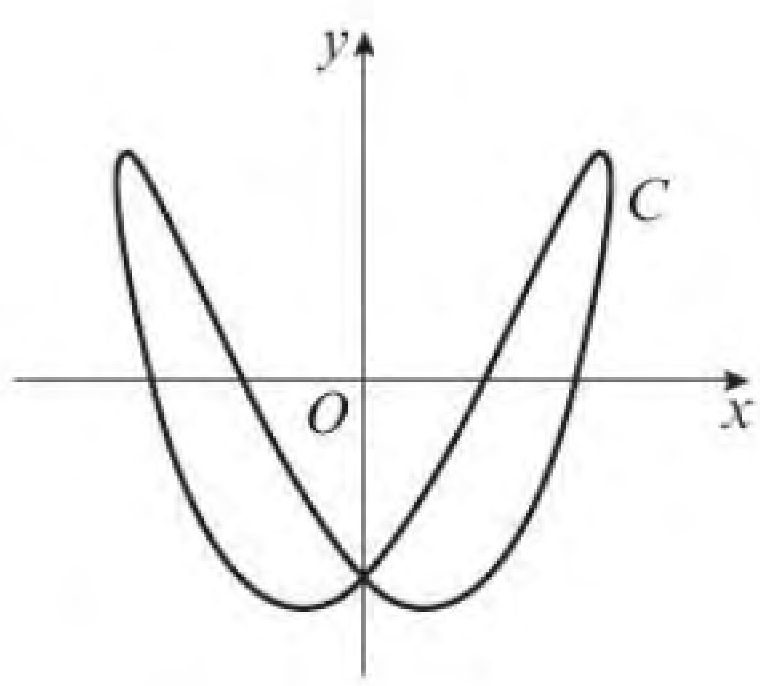
Here is a sketch of the curve C and the tangent at the point where $t = \frac{\pi}{3}$:



Now try this

The diagram shows a sketch of the curve C with parametric equations

$x = 2\cos\left(t - \frac{\pi}{3}\right),\ y = 3\sin 2t, \qquad 0 \leqslant t < 2\pi$



(a) Find $\frac{dy}{dx}$ in terms of t. Answer **(4 marks)**

(b) Find an equation of the normal to the curve at the point where $t = 0$. Answer **(4 marks)**

(c) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$. **(5 marks)** Answer

Find all the values of t where $\frac{dy}{dx} = 0$, then find the x- and y-coordinates at each of these points.