Summary of key points

7 For a random sample of size n taken from a random variable $X \sim \mathrm{N}(\mu, \sigma^2)$, the sample mean, $\overline{X}$, is normally distributed with $\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$.

8 For the sample mean of a normally distributed random variable, $\overline{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$,

$Z = \dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ is a normally distributed random variable with $Z \sim \mathrm{N}(0, 1)$.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Normal hypothesis testing

You can use a **sample** from a normally distributed population to test hypotheses about the mean of that population. Your test statistic will be the **sample mean**, which is written as $\overline{X}$. For a normally distributed population, the sample mean is also normally distributed. It has the same mean as the population but a **different variance**. Use the rule on the right to work out its distribution.

Golden rule

If a random sample of size n is taken from a normally distributed population, $X \sim N(\mu, \sigma^2)$, then the sample mean has distribution

$$\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Worked example

The masses of the loaves of bread made in a particular bakery are normally distributed with standard deviation 30 g.

The bakery claims the mean mass is 800 g. A consumer group believes the actual average mass is less than this, and takes a random sample of 20 loaves from the bakery. The mean mass of the loaves in the sample was 788 g.

Test the consumer group's claim, stating your hypotheses clearly and using a 5% significance level. **(4 marks)**

H_0: $\mu = 800$, H_1: $\mu < 800$

Let X represent the mass of a loaf of bread and assume H_0 to be true, so that $X \sim N(800, 30^2)$

So for the sample mean:

$\overline{X} \sim N\left(800, \frac{30^2}{20}\right)$ or $\overline{X} \sim N(800, 45)$

$P(\overline{X} < 788) = 0.0368$

$0.0368 < 0.05$ so there is evidence to reject H_0 at the 5% level, and conclude that the mean mass of loaves in the whole population is less than 800 g.

Worked example

A random sample of 8 observations is taken from the random variable $X \sim N(\mu, 4^2)$, and is used to test H_0: $\mu = 18$ against H_1: $\mu \neq 18$ at the 5% level.

(a) Find the critical region for the sample mean, $\overline{X}$. **(3 marks)**

Assume H_0 to be true, so that $X \sim N(18, 4^2)$

So for the sample mean:

$\overline{X} \sim N\left(18, \frac{4^2}{8}\right)$ or $\overline{X} \sim N(18, 2)$

$P(\overline{X} < a) = 0.025$ gives $a = 15.228$

$P(\overline{X} > b) = 0.025$ gives $b = 20.772$

The critical region is $\overline{X} < 15.228$ or $\overline{X} > 20.772$

The values observed in the sample are:

12.1	17.3	18.3	10.6
11.5	19.1	17.5	20.9

(b) Comment on this sample in light of the critical region. **(2 marks)**

$\overline{x} = \frac{\Sigma x}{n} = \frac{127.3}{8} = 15.9$ (1 d.p.)

This value lies outside the critical region, so there is not sufficient evidence to reject H_0 at the 5% level.

This is a **two-tailed test** so you need to halve the probability in each tail.

Now try this

A computer-controlled milling machine produces metal circles with a diameter which is normally distributed with mean 25 mm and standard deviation 1.2 mm.

After being serviced, the machine continues to produce circles with a diameter which is normally distributed with a standard deviation of 1.2 mm. However, a technician believes that the mean diameter of circles produced by the machine has increased.

To test his theory, he takes a random sample of 15 circles.

(a) Find the critical region for his test, using a significance level of 1% and stating your hypotheses clearly. Answer **(4 marks)**

The mean diameter of the circles in the sample is found to be 26.1 mm.

(b) Comment on this observation in light of your answer to part (a). Answer **(1 mark)**