Summary of key points

1 You can use **proof by induction** to prove that a general statement is true for all positive integers.

2 Proof by mathematical induction usually consists of the following four steps:

- **Basis:** Show the general statement is true for $n = 1$.
- **Assumption:** Assume that the general statement is true for $n = k$.
- **Inductive:** Show the general statement is true for $n = k + 1$.
- **Conclusion:** State that the general statement is then true for all positive integers, n.

Notation $\mathbb{Z}^+$ is the set of **positive integers**, 1, 2, 3, …. It is equivalent to $\mathbb{N}$, the set of natural numbers.

Hint $\sum_{r=1}^{n}(-1)^r r^2 = -1^2 + 2^2 - 3^2 + 4^2 - 5^2 + \dots$

As both $f(k)$ and $3(k^2 + k - 2)$ are divisible by 3 then their sum must also be divisible by 3.

Problem-solving

When proving that an expression $f(n)$ is divisible by r, you can complete the induction step by showing that $f(k + 1) - f(k)$ is divisible by r.

Problem-solving

Always keep an eye on what you are trying to prove. You need to show that this expression is divisible by 133, so write $143(12^{2k-1})$ as $10(12^{2k-1}) + 133(12^{2k-1})$.

This is the right-hand side of the original equation with n replaced by $k + 1$.