Summary of key points

1 $\mathbf{i} = \sqrt{-1}$ and $\mathbf{i}^2 = -1$

2 An **imaginary number** is a number of the form bi, where $b \in \mathbb{R}$.

3 A **complex number** is written in the form $a + bi$, where $a, b \in \mathbb{R}$.

Notation The set of all complex numbers is written as $\mathbb{C}$.
For the complex number $z = a + bi$:
- $\text{Re}(z) = a$ is the real part
- $\text{Im}(z) = b$ is the imaginary part

Notation Complex numbers are often represented by the letter z or the letter w.

4 Complex numbers can be added or subtracted by adding or subtracting their real parts and adding or subtracting their imaginary parts.

5 You can multiply a real number by a complex number by multiplying out the brackets in the usual way.

6 If $b^2 - 4ac < 0$ then the quadratic equation $ax^2 + bx + c = 0$ has two distinct complex roots, neither of which is real.

7 For any complex number $z = a + bi$, the **complex conjugate** of the number is defined as $z^* = a - bi$.

$z + z^* = 2a$
$zz^* = a^2 + b^2$

Write $\dfrac{5 + 4i}{2 - 3i}$ in the form $a + bi$.

$$\frac{5 + 4i}{2 - 3i} = \frac{5 + 4i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i}$$

The complex conjugate of the denominator is $2 + 3i$. Multiply both the numerator and the denominator by the complex conjugate.

8 For real numbers a, b and c, if the roots of the quadratic equation $az^2 + bz + c = 0$ are non-real complex numbers, then they occur as a conjugate pair.

Note If z_1 is real, then $z_1{}^* = z_1$.

9 If the roots of a quadratic equation are α and β, then you can write the equation as $(z - \alpha)(z - \beta) = 0$ or $z^2 - (\alpha + \beta)z + \alpha\beta = 0$.

Notation Roots of complex-valued polynomials are often written using Greek letters such as α (alpha), β (beta) and γ (gamma).

10 If f(z) is a polynomial with real coefficients, and z_1 is a root of f(z) = 0, then $z_1{}^*$ is also a root of f(z) = 0.

11 An equation of the form $az^3 + bz^2 + cz + d = 0$ is called a cubic equation, and has three roots.
For a cubic equation with real coefficients, either:
- all three roots are real, or
- one root is real and the other two roots form a complex conjugate pair.

12 An equation of the form $az^4 + bz^3 + cz^2 + dz + e = 0$ is called a quartic equation, and has four roots.
For a quartic equation with real coefficients, either:
- all four roots are real, or
- two roots are real and the other two roots form a complex conjugate pair, or
- two roots form a complex conjugate pair and the other two roots also form a complex conjugate pair.

Problem-solving Use the factor theorem to help: if f(α) = 0, then α is a root of the polynomial and $z - \alpha$ is a factor of the polynomial.

Watch out You could use your calculator to solve $z^2 - 2z + 17 = 0$. However, you should still write down the equation you are solving, and both roots.

Problem-solving It is possible to factorise a polynomial without using a formal algebraic method. Here, the polynomial is factorised by 'inspection'. By considering each term of the quartic separately, it is possible to work out the missing coefficients.