Summary of key points

1 A vector equation of a straight line passing through the point A with position vector **a**, and parallel to the vector **b**, is

$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$

where λ is a scalar parameter.

2 A vector equation of a straight line passing through the points C and D, with position vectors **c** and **d** respectively, is

$\mathbf{r} = \mathbf{c} + \lambda(\mathbf{d} - \mathbf{c})$

where λ is a scalar parameter.

3 If $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, the equation of the line with vector equation $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ can be given in Cartesian form as:

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}$$

Each of these three expressions is equal to λ.

4 The vector equation of a plane is

$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where:

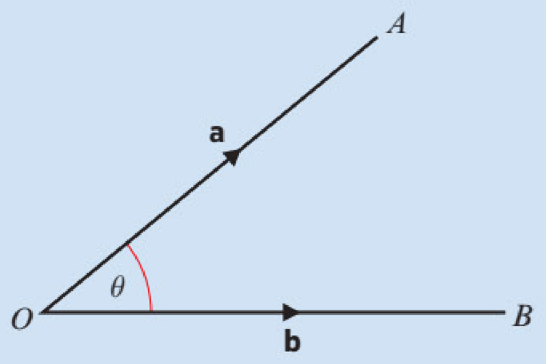
- **r** is the position vector of a general point in the plane
- **a** is the position vector of a point in the plane
- **b** and **c** are non-parallel, non-zero vectors in the plane
- λ and μ are scalars

5 A Cartesian equation of a plane in three dimensions can be written in the form $ax + by + cz = d$ where a, b, c and d are constants, and $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the normal vector to the plane.

6 The **scalar product** of two vectors **a** and **b** is written as **a.b** (say '**a** dot **b**'), and defined as

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

where θ is the angle between **a** and **b**.



7 If **a** and **b** are the position vectors of the points A and B, then $\cos(\angle AOB) = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

8 The non-zero vectors **a** and **b** are perpendicular if and only if $\mathbf{a}.\mathbf{b} = 0$.

9 If **a** and **b** are parallel, $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|$. In particular, $\mathbf{a}.\mathbf{a} = |\mathbf{a}|^2$.

10 If $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$

$$\mathbf{a}.\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}.\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$$

11 The acute angle θ between two intersecting straight lines is given by

$$\cos\theta = \left|\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right|$$

where **a** and **b** are direction vectors of the lines.

12 The scalar product form of the equation of a plane is $\mathbf{r}.\mathbf{n} = k$ where $k = \mathbf{a}.\mathbf{n}$ for any point in the plane with position vector **a**.

13 The acute angle θ between the line with equation $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and the plane with equation $\mathbf{r}.\mathbf{n} = k$ is given by the formula

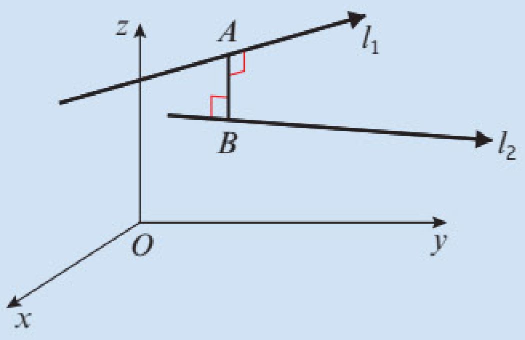
$$\sin\theta = \left|\frac{\mathbf{b}.\mathbf{n}}{|\mathbf{b}||\mathbf{n}|}\right|$$

14 The acute angle θ between the plane with equation $\mathbf{r}.\mathbf{n}_1 = k_1$ and the plane with equation $\mathbf{r}.\mathbf{n}_2 = k_2$ is given by the formula

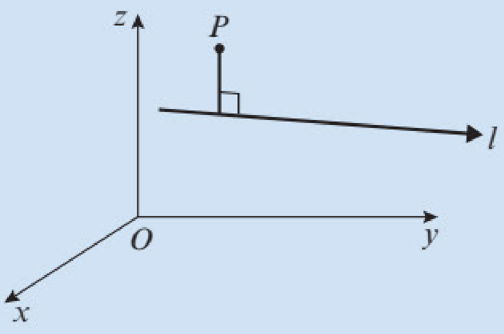
$$\cos\theta = \left|\frac{\mathbf{n}_1.\mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right|$$

15 Two lines are **skew** if they are not parallel and they do not intersect.

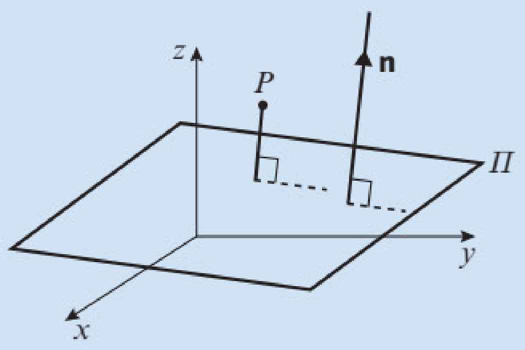
16 For any two non-intersecting lines l_1 and l_2 there is a unique line segment AB such that A lies on l_1, B lies on l_2 and AB is perpendicular to both lines.



17 The perpendicular from a point P to a line l is a line drawn from P at right angles to l.



18 The perpendicular from a point P to a plane Π is a line drawn from P parallel to the normal vector **n**.



19 k is the length of the perpendicular from the origin to a plane Π, where the equation of plane Π is written in the form $\mathbf{r}.\hat{\mathbf{n}} = k$, where $\hat{\mathbf{n}}$ is a unit vector perpendicular to Π.

The perpendicular distance from the point with coordinates (α, β, γ) and the plane with equation $ax + by + cz = d$ is

$$\frac{|a\alpha + b\beta + c\gamma - d|}{\sqrt{a^2 + b^2 + c^2}}$$