Summary of key points

6 You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\frac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

7 The force of **gravity** causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.

8 An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\text{ m s}^{-2}$.

In a velocity–time graph the **gradient** represents the acceleration.

If the velocity–time graph is a straight line, then the acceleration is constant.

5 The area between a velocity–time graph and the horizontal axis represents the distance travelled.

For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.

Constant acceleration 1

There are **five** formulae which describe the motion of a particle travelling in a **straight line** with constant acceleration. They are given in the formulae booklet but you should learn them. The other three formulae are covered on the next page.

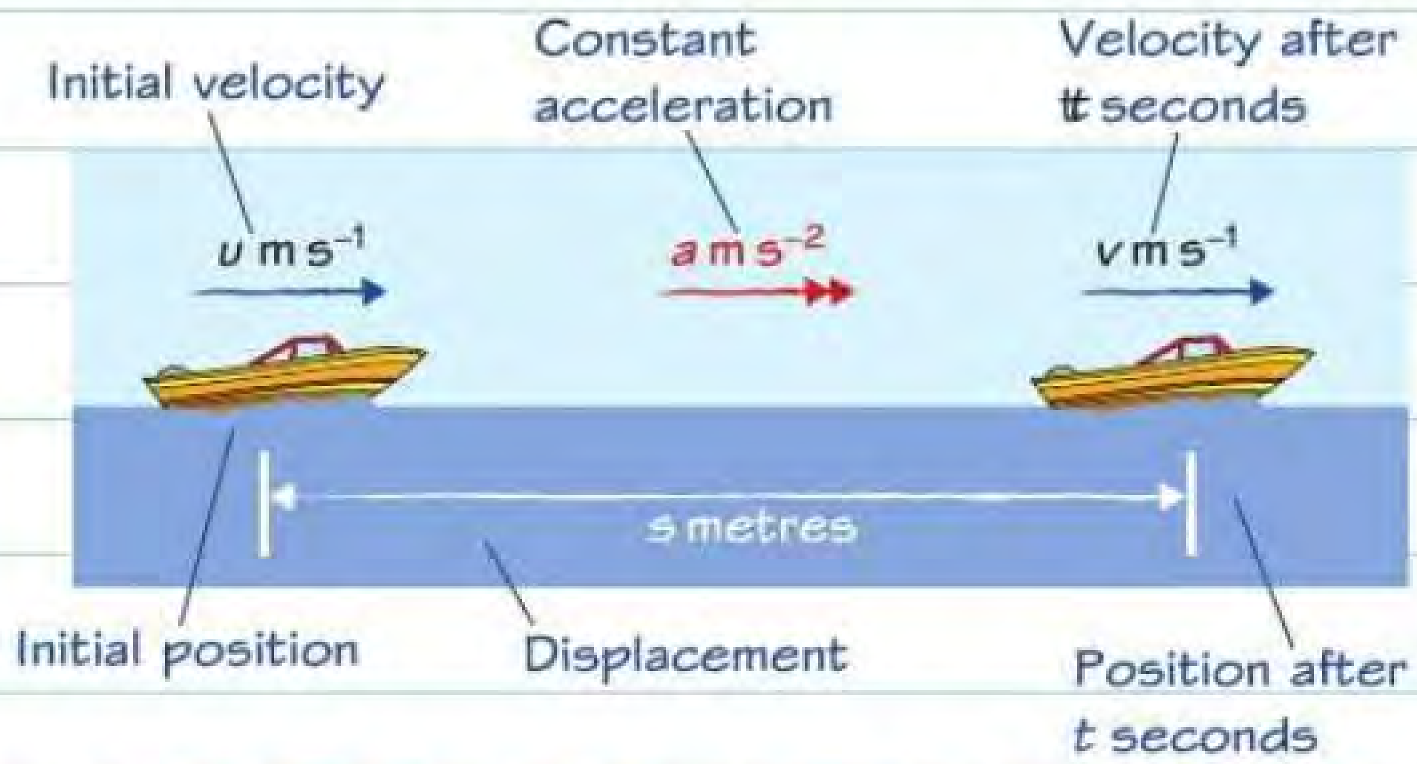
Constant acceleration formulae

Here are the first two formulae you need to learn for motion in a straight line with constant acceleration.

1 $v = u + at$

2 $s = \frac{1}{2}(u + v)t$

Look at the diagram on the right to see what each letter represents.

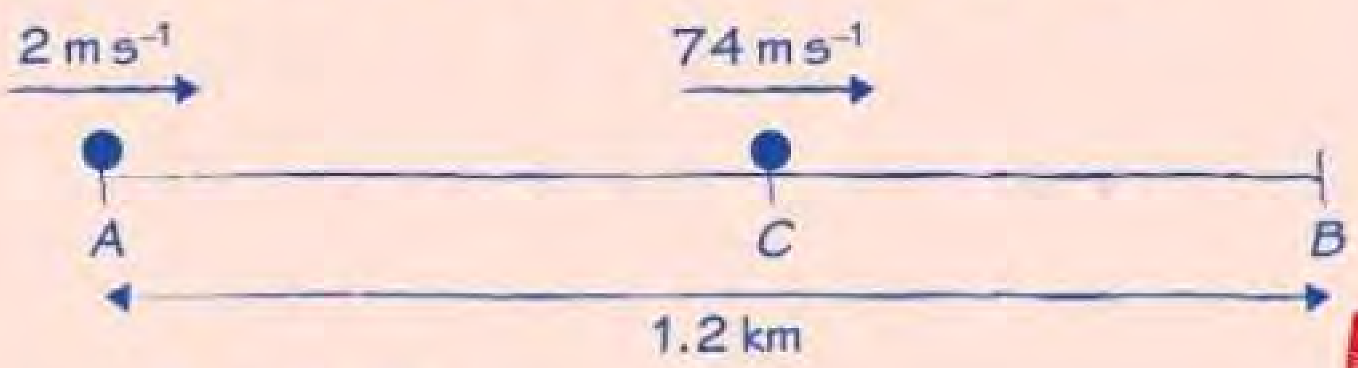


Worked example

In taking off, an aircraft moves on a straight runway AB of length 1.2 km. The aircraft moves from A with initial speed 2 m s^{-1}. It moves with constant acceleration and 20 s later it leaves the runway at C with speed 74 m s^{-1}. Find

(a) the acceleration of the aircraft **(2 marks)**

$\cancel{s = ?}$, $u = 2$, $v = 74$, $a = ?$, $t = 20$



$v = u + at$

$74 = 2 + a \times 20$

$20a = 72$

$a = 3.6$ m s^{-2}

(b) the distance CB. **(4 marks)**

$s = ?$, $u = 2$, $v = 74$, $\cancel{a = 3.6}$, $t = 20$

$s = \frac{1}{2}(u + v)t$

$AC = \frac{1}{2}(2 + 74) \times 20$

$= 760$ m

$CB = 1200$ m $- 760$ m $= 440$ m

Using *SUVAT*

The constant acceleration formulae are sometimes called the *SUVAT* formulae. In the exam you should write down all five letters.

- ✓ Write in any values you **know**.
- ✓ Put a **question mark** next to the value you want to find.
- ✓ **cross out** any values you don't need for that question.

This will help you choose which formula to use.

When you are using the ***suvat*** formulae **always** write down the formula **before** you substitute in.

Problem solved!

Read the question carefully. The distance AB is 1.2 km, but the aircraft does not take off at B. If you have to solve a constant acceleration question involving **three points** like this one, it's a good idea to draw a quick sketch to help you see what is going on. In part (b), s is the distance AC in metres. You need to subtract it from 1200 m to find the distance CB.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

A car moves along a straight stretch of road AB. The car moves with initial speed 2 m s^{-1} at point A. It accelerates constantly for 12 seconds, reaching a speed of 23 m s^{-1} at point B. Find

Answer (a) the acceleration of the car **(2 marks)**

Answer (b) the distance AB. **(2 marks)**

You can answer both parts of this question using the formulae given on this page. But you can use any of the other ***suvat*** formulae if you are confident with them.

Constant acceleration 2

Here are three more formulae you can use to solve problems involving constant acceleration in a **straight line**. Together with the formulae on page 82, these are called the *SUVAT* formulae.

1 $v^2 = u^2 + 2as$

2 $s = ut + \frac{1}{2}at^2$

3 $s = vt - \frac{1}{2}at^2$

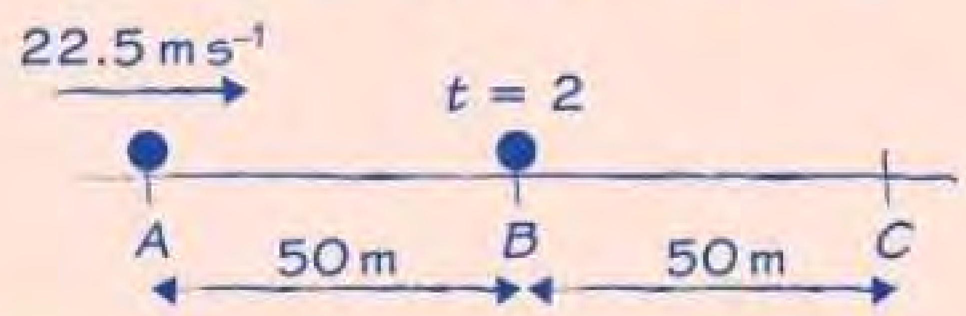
For a definition of what each letter represents, look at the diagram on page 82.

Worked example

A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points A, B and C, where $AB = 50$ m and $BC = 50$ m. The front of the train passes A with speed 22.5 m s^{-1}, and 2 s later it passes B. Find

(a) the acceleration of the train **(3 marks)**

$s = 50$, $u = 22.5$, $\cancel{v = ?}$, $a = ?$, $t = 2$



$s = ut + \frac{1}{2}at^2$

$50 = 22.5 \times 2 + \frac{1}{2} \times a \times 2^2$

$50 = 45 + 2a$

$a = 2.5$ m s^{-2}

(b) the speed of the front of the train when it passes C **(3 marks)**

$s = 100$, $u = 22.5$, $v = ?$, $a = 2.5$, $\cancel{t = ?}$

$v^2 = u^2 + 2as$

$v^2 = 22.5^2 + 2 \times 2.5 \times 100$

$= 1006.25$

$v = 31.72... = 31.7$ m s^{-1} (3 s.f.)

(c) the time that elapses from the instant the front of the train passes B to the instant it passes C. **(4 marks)**

$s = 50$, $u = ?$, $v = 31.7214...$, $a = 2.5$, $t = ?$

$s = vt - \frac{1}{2}at^2$

$50 = 31.72... \times t - \frac{1}{2} \times 2.5 \times t^2$

$1.25t^2 - 31.72...t + 50 = 0$

$$t = \frac{-(-31.72...) \pm \sqrt{(-31.72...)^2 - 4 \times 1.25 \times 50}}{2.5}$$

$t = 1.688...$ or $\cancel{t = 23.688...}$

The time elapsed is 1.69 s. (3 s.f.)

Units

You need to make sure that your measurements are in the correct units.

- t (time) is measured in seconds
- s (displacement) is measured in metres
- u and v (velocity) is measured in m s^{-1}

m s^{-1} means m/s or 'metres per second'

- a (acceleration) is measured in m s^{-2}

m s^{-2} means m/s^2, 'metres per second squared', or 'metres per second per second'

Problem solved!

In part (c) you get two solutions to the quadratic equation. You know the train is travelling forwards at B and accelerating. It will take less than 2 seconds to travel from B to C, so choose $t = 1.688...$

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

A boat travels in a straight line with constant deceleration, between two buoys A and B, which are a distance 300 m apart. The boat passes buoy A with initial speed 16 m s^{-1}, and passes buoy B 30 seconds later. Find

(a) the deceleration of the boat
Answer **(3 marks)**

(b) the speed of the boat as it passes B Answer **(4 marks)**

(c) the distance from B at which the boat comes to rest. **(4 marks)**

Answer

Motion under gravity

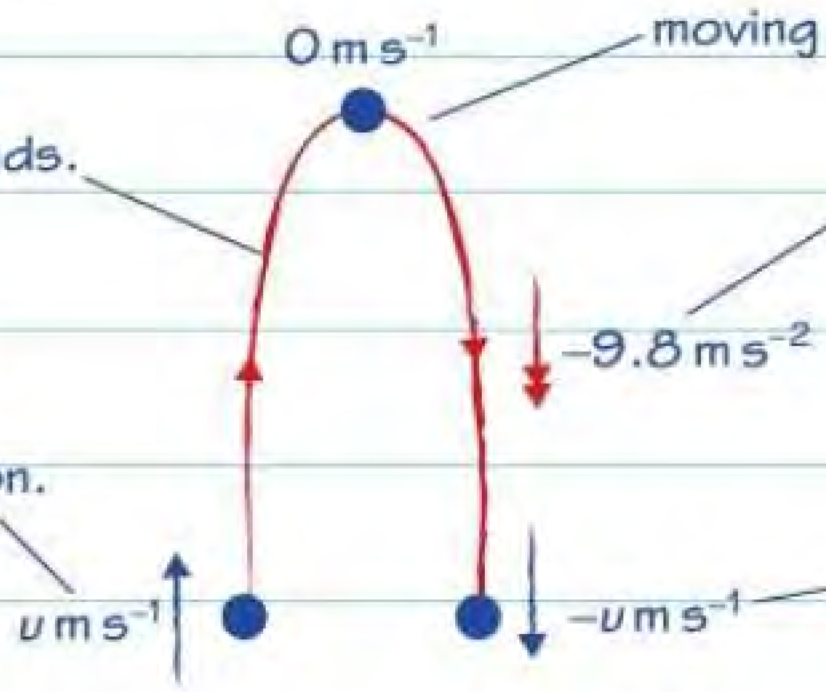
If you ignore **air resistance**, you can model an object moving freely under gravity as a particle with **constant downward acceleration**. This means you can use the constant acceleration formulae given on pages 82 and 83.

Up and down

Here are some useful facts about particles moving vertically under gravity.

If the particle is thrown upwards and takes t seconds to reach its **maximum height** then it returns to its starting position after $2t$ seconds.

You can choose which direction is positive in a question. You usually want UP to be the positive direction.



At its **maximum height**, a vertically moving particle has **zero** velocity.

UP is the positive direction, so the acceleration due to gravity is **negative**. It remains **constant** throughout. Show acceleration with a double arrow so you don't confuse it with velocity.

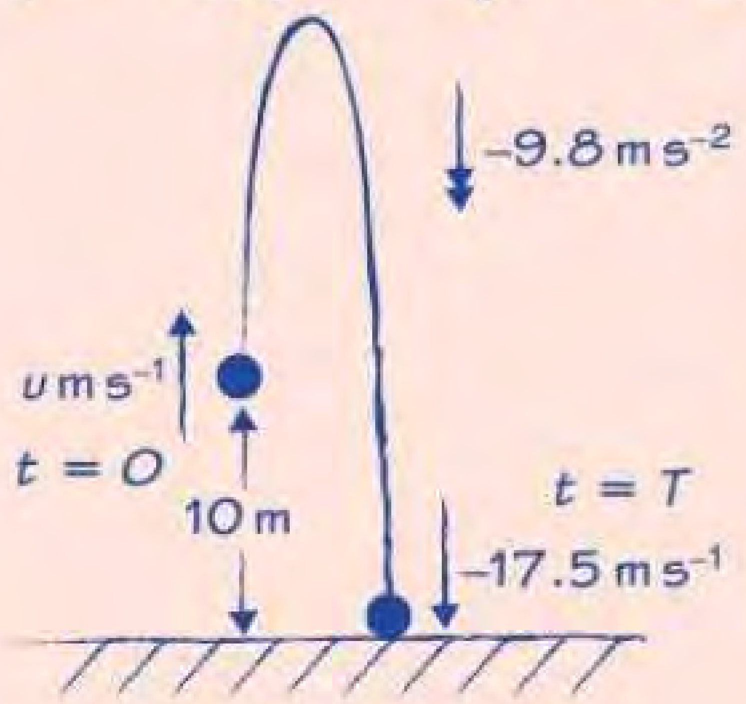
When the particle returns to its initial position, it has the **same speed** as it started with but in the **opposite direction**.

Worked example

At time $t = 0$, a particle is projected vertically upwards with speed $u\,\text{m s}^{-1}$ from a point 10 m above the ground. At time T seconds, the particle hits the ground with speed $17.5\,\text{m s}^{-1}$. Find

(a) the value of u **(3 marks)**

$s = -10,\ u = ?,\ v = -17.5,\ a = -9.8,\ \cancel{t = T}$



The **positive** direction is **up**, so a and v are **negative**. The finishing position is 10 m **below** the starting position, so s is negative as well.

$$v^2 = u^2 + 2as$$
$$(-17.5)^2 = u^2 + 2 \times (-9.8) \times (-10)$$
$$306.25 = u^2 + 196$$
$$u^2 = 110.25$$
$$u = 10.5 = 11\,\text{m s}^{-1}\ (2\text{ s.f.})$$

(b) the value of T. **(4 marks)**

$\cancel{s = -10},\ u = 10.5,\ v = -17.5,\ a = -9.8,\ t = T$

$$v = u + at$$
$$-17.5 = 10.5 + (-9.8) \times T$$
$$9.8T = 28$$
$$T = 2.857... = 2.9\text{ s}\ (2\text{ s.f.})$$

You've used $g = 9.8\,\text{m s}^{-2}$ so round to 2 s.f.

Accuracy and gravity

Unless you're told otherwise, you should use this value for the acceleration due to gravity:

$$g = 9.8\,\text{m s}^{-2}$$

This value for g is correct to 2 **significant figures**.

This means that if you use $g = 9.8\,\text{m s}^{-2}$ in your calculation you should give your final answer correct to 2 significant figures.

Don't use rounded values in calculations. In the Worked example on the left, you can give the answer to part (a) as $11\,\text{m s}^{-1}$, but you still need to use $u = 10.5$ in part (b).

Now try this

A diver projects herself upwards from a diving platform with a speed of $3.5\,\text{m s}^{-1}$. The platform is 15 m above the water. Modelling the diver as a particle moving freely under gravity, find

(a) the greatest height above the water reached by the diver Answer **(4 marks)**

(b) the speed with which the diver hits the water **(3 marks)** Answer

(c) the total time from when the diver leaves the platform to when she hits the water. **(3 marks)**

Answer

Had a look ☐ Nearly there ☐ Nailed it! ☐

Deriving *suvat* equations

You can use **integration** to derive the *suvat* equations for motion with constant acceleration.

Worked example

A particle moves in a straight line with constant acceleration, $A\,\text{m s}^{-2}$. At time t seconds the velocity of the particle is $v\,\text{m s}^{-1}$ and its displacement relative to a fixed origin is s m.

Given that the initial velocity of the particle is $U\,\text{m s}^{-1}$ and its initial displacement is 0 m, prove that

(a) $v = U + At$ **(3 marks)**

$v = \int A\,dt$

$= At + c$

When $t = 0$, $v = U \Rightarrow c = U$

So $v = At + U = U + At$ as required

(b) $s = Ut + \frac{1}{2}At^2$ **(3 marks)**

$s = \int v\,dt = \int (U + At)dt$

$= Ut + \frac{1}{2}At^2 + c$

When $t = 0$, $s = U \Rightarrow c = 0$

So $s = Ut + \frac{1}{2}At^2$ as required

In this example capital letters are used to represent letters that can be treated as constants when you integrate.

Integrate, then use the given initial conditions to find the constants of integration. Remember to write your final answers in the form asked for in the question.

Worked example

Given that

$v = u + at$ ①

$s = ut + \frac{1}{2}at^2$ ②

prove that $v^2 = u^2 + 2as$. You may not make use of any formulae other than those given above. **(3 marks)**

Squaring both sides of ①:

$v^2 = (u + at)^2$

$= u^2 + 2uat + a^2t^2$

$= u^2 + 2a(ut + \frac{1}{2}at^2)$

Substituting ② into the above:

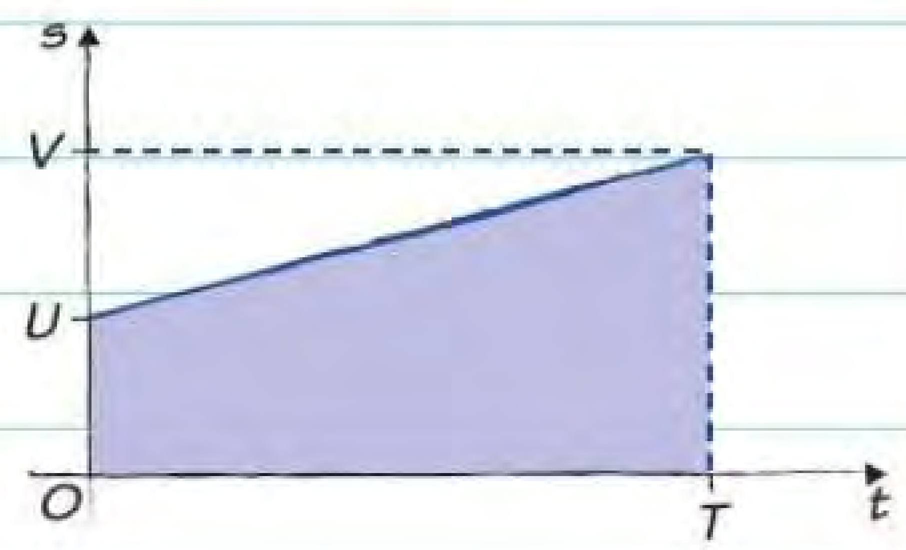
$v^2 = u^2 + 2as$ as required.

You can use the two *suvat* formulae in the example above left to derive the other *suvat* formulae. Have a look at pages 82 and 83 for a reminder about the five *suvat* formulae.

Using a velocity–time graph

You can also derive the *suvat* formulae from a velocity–time graph. The diagram shows a velocity–time graph for a particle accelerating constantly from velocity $U\,\text{m s}^{-1}$ to velocity $V\,\text{m s}^{-1}$ in time T seconds. The acceleration, $a\,\text{m s}^{-2}$, of the particle is equal to the gradient of the graph:

$a = \frac{V - U}{T}$, so $V = U + aT$



Now try this

1 A particle moves in a straight line along the x-axis. At time t seconds, its distance from the origin is given by

$s = 10t - kt^2,\ t \geqslant 0$

where k is a constant.

(a) Show that the particle is moving with constant acceleration. **(4 marks)**

Answer

(b) Given that the particle is instantaneously at rest when $t = 4$, find the value of k.

Answer **(3 marks)**

2 Use the velocity–time graph above to show that the distance travelled, s m, by the particle in time T seconds is given by

$s = \left(\frac{U + V}{2}\right)T$ Answer **(4 marks)**