Summary of key points

9 If $y = ax^n$ then the graph of $\log y$ against $\log x$ will be a straight line with gradient n and vertical intercept $\log a$.

10 If $y = ab^x$ then the graph of $\log y$ against x will be a straight line with gradient $\log b$ and vertical intercept $\log a$.

Modelling with logs

You can use logarithms to determine the constants in some **exponential** and **polynomial** models. There are two different cases that you need to know about:

1 The polynomial model $y = ax^n$

In this model a and n are constants. If x and y satisfy this model then the graph of **log y against log x** will be a straight line: $\log y = n \log x + \log a$

gradient = n y-intercept = $\log a$

2 The exponential model $y = kb^x$

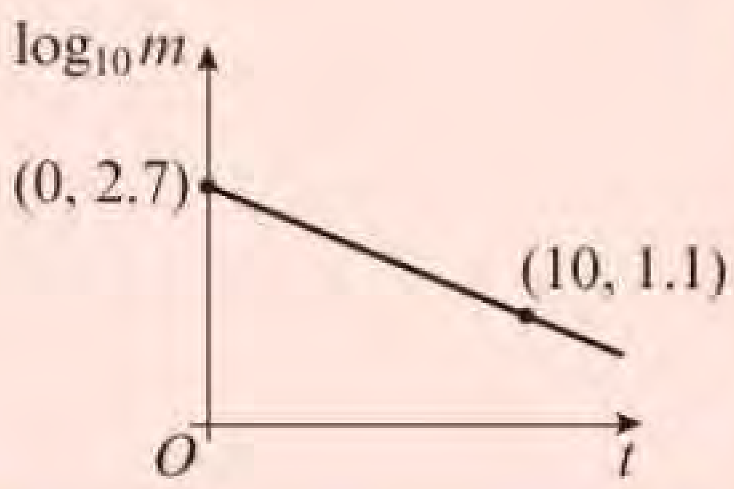
In this model k and b are constants. If x and y satisfy this model then the graph of **log y against x** will be a straight line: $\log y = (\log b)\,x + \log a$

gradient = $\log b$ y-intercept = $\log a$

Worked example

A scientist models the mass, m grams, of a fluorine sample and the time elapsed, t hours, using the equation $m = pq^t$, where p and q are constants.

She observes the actual mass over a period of 10 hours, and plots the graph shown on the right, of t against $\log m$.



(a) Find an equation for the line. **(2 marks)**

Gradient $= \dfrac{1.1 - 2.7}{10} = -0.16$

$\log m = -0.16t + 2.7$

(b) Determine the values of p and q in the model to 3 significant figures. **(4 marks)**

$m = 10^{-0.16t + 2.7}$

$= 10^{2.7} \times (10^{-0.16})^t$

$= 501 \times 0.692^t$

So $p = 501$ and $q = 0.692$ (3 s.f.)

For part (b), the safest approach is to rearrange the equation of the line into the form $m = pq^t$, then compare values.

Check it!

$501 \times 0.692^{10} = 12.6$

$\log 12.6 = 1.1$ ✓

(c) Interpret these values in the context of the model. **(2 marks)**

$p = 501$ is the initial mass of the sample in grams.

$q = 0.692$ is the proportional change in the sample each hour.

(d) Use the model to predict the mass of the sample after 3 days. **(1 mark)**

3 days = 72 hours, so $m = 501 \times 0.692^{72} = 1.51 \times 10^9$ g

(e) Give one reason why this prediction may not be accurate. **(1 mark)**

The model is based on 10 hours of data, so it may not be accurate over a longer period.

Now try this

A computer algorithm is used to allocate medical students to hospitals. When there are N students, the runtime of the algorithm, x milliseconds, is expected to follow the rule $x = aN^b$, where a and b are constants.

(a) Show that this relationship can be written in the form $\log x = k \log N + c$, giving k and c in terms of a and b. **(2 marks)** Answer

The algorithm is run a number of times and the following values of x and N are found:

N	1000	1500	2000	2500	3000	3500	4000
x	460	980	1660	2510	3520	4680	5990

(b) Plot a graph of $\log x$ against $\log N$, and comment on the accuracy of the expected model. **(3 marks)** Answer

(c) Find the values of a and b, giving your answers to 2 significant figures. **(4 marks)** Answer