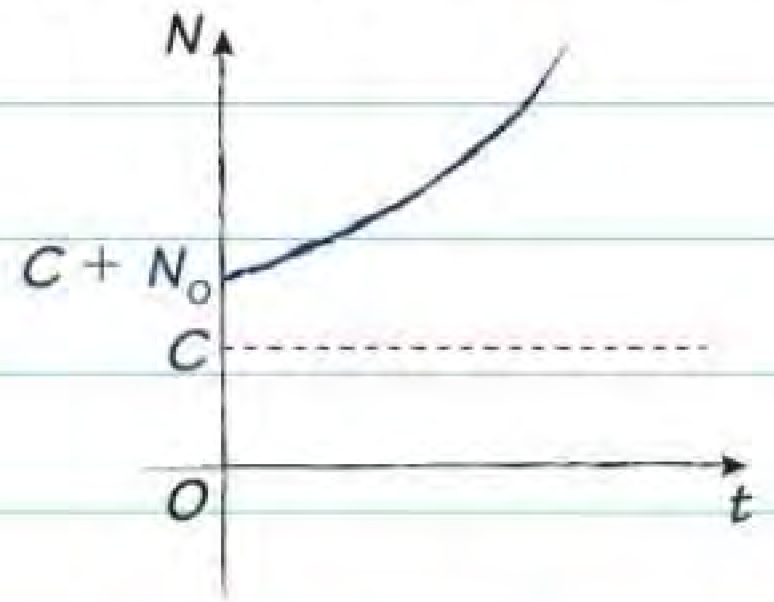
# Exponential modelling

You can use exponential functions to model lots of real-life situations.

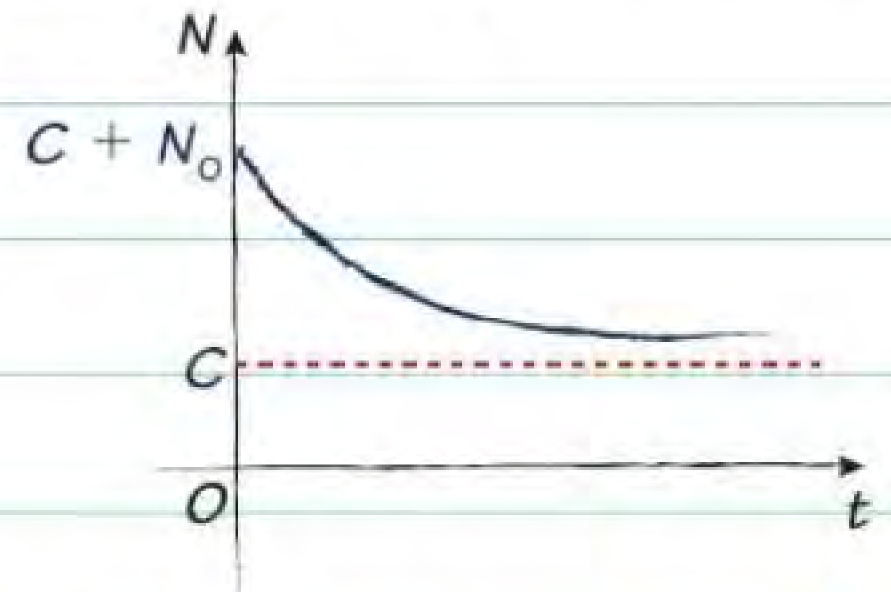
## 1 Growth models

A typical growth model can be described as $N = C + N_0\,e^{kt}$




## 2 Decay models

A typical decay model can be described as $N = C + N_0\,e^{-kt}$




## Worked example

This model describes the temperature $P$ °C of the water in a kettle $t$ minutes after it has boiled:

$$P = 25 + A\,e^{-kt}, \quad t \geqslant 0$$

where $A$ and $k$ are positive constants.

(a) Given that the initial temperature of the water was 100 °C, find the value of $A$. **(2 marks)**

When $t = 0$, $P = 25 + Ae^0$

$100 = 25 + A$

$A = 75$

After 10 minutes, the water in the kettle has cooled down to 40 °C.

(b) Show that $k = \frac{1}{10}\ln 5$. **(3 marks)**

$40 = 25 + 75e^{-10k}$

$15 = 75e^{-10k}$

$\frac{1}{5} = e^{-10k}$

$-10k = \ln\left(\frac{1}{5}\right) = \ln(5^{-1}) = -\ln 5$

So $k = \frac{1}{10}\ln 5$

(c) Find the temperature of the water after 18 minutes, in °C to 1 decimal place. **(2 marks)**

$P = 25 + 75e^{-\left(\frac{1}{10}\ln 5\right)\times 18} = 29.1$°C (1 d.p.)

You will sometimes be given a complete exponential model and asked to use it. In this example, you are not given two of the constants in the model. To find them:

- Substitute the information given in the question into the equation of the model. Make sure you substitute the right variable in the right place.
- Solve an equation to find the unknown constants.

### Rates of change

You might have to find the **rate of change** of a quantity in an exponential model. You can do this by **differentiating** with respect to time. In the model in the Worked example on the left, the rate of change of temperature with time would be given by

$$\frac{dP}{dt} = -kAe^{-kt}$$

The units would be °C/min. The fact that the rate of change is **negative** tells you that the temperature is **decreasing**.

## Now try this

1 The number of cells, $N$, in a bacterial culture at a time $t$ hours after midday is modelled as

$$N = 100\,e^{0.8t}, \quad t \geqslant 0$$

(a) Write down the number of cells in the culture at midday. Answer **(1 mark)**

(b) Show that the rate of change of $N$ at time $t$ is directly proportional to the size of the population. Answer **(3 marks)**

2 The mass, $M$ grams, of a sample of radon after $t$ hours is modelled using the equation

$$M = 250\,e^{-kt}, \quad t \geqslant 0$$

where $k$ is a positive constant.

(a) What was the initial mass of the sample? Answer **(1 mark)**

After 90 hours the sample has lost half its mass.

(b) Find the value of $k$ to 3 significant figures. Answer **(4 marks)**