Had a look ☐ Nearly there ☐ Nailed it! ☐

Differentiation 1

You can **differentiate** a function to find its **derivative** or **gradient function**.

The derivative is written as f′(x) or $\frac{dy}{dx}$

Differentiating x^n

$y = f(x)$ $\quad y = x^n$

Differentiation — Multiply by the power …

$\frac{dy}{dx} = f'(x)$ $\quad \frac{dy}{dx} = nx^{n-1}$ … then reduce the power by 1

This rule works for **any** value of *n*, including **fractions** and **negative** numbers.

Golden rules

1. Write every term in a polynomial in the form ax^n **before** differentiating.
   $\sqrt{x} \rightarrow x^{\frac{1}{2}}$ $\quad \frac{6}{x^2} \rightarrow 6x^{-2}$
2. Constant terms differentiate to **zero**, and *x* terms differentiate to a **constant**.
   $f(x) = 7 \rightarrow f'(x) = 0$ $\quad f(x) = 3x + 1 \rightarrow f'(x) = 3$

Worked example

Given that $y = 3x^6 - 8 + \frac{1}{x^3}$, $x \neq 0$, find $\frac{dy}{dx}$ **(3 marks)**

$y = 3x^6 - 8 + x^{-3}$

$\frac{dy}{dx} = 18x^5 - 3x^{-4}$

Start by rewriting $\frac{1}{x^3}$ as x^{-3}.
Remember that you are multiplying by −3 when you differentiate this term, so the new term is **negative.**

Worked example

Differentiate with respect to x

(a) $x^3 - 3\sqrt{x} + \frac{x}{7}$ **(3 marks)**

$f(x) = x^3 - 3x^{\frac{1}{2}} + \frac{1}{7}x$

$f'(x) = 3x^2 - \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{7}$

(b) $\frac{kx + 5}{x^2}$ **(3 marks)**

$f(x) = \frac{kx}{x^2} + \frac{5}{x^2} = kx^{-1} + 5x^{-2}$

$f'(x) = -kx^{-2} - 10x^{-3}$

With respect to *x* just means that ***x*** is the variable. You should treat any other letters in the function as **constants**.

It's OK to leave powers as **negative numbers** or **fractions** in your final answers. The answer to part (a) could be in either of these forms:

$3x^2 - \frac{3}{2}x^{-\frac{1}{2}} + \frac{1}{7}$ $\quad 3x^2 - \frac{3}{2\sqrt{x}} + \frac{1}{7}$

Now try this

1 Given that $y = \frac{(x+3)^2}{x}$, $x \neq 0$, find $\frac{dy}{dx}$ **(4 marks)** Answer

Multiply out the brackets, then write the function in the form $y = ax + b + cx^{-1}$ before differentiating.

2 (a) Write $\frac{2 + 5\sqrt{x}}{x}$ in the form $2x^p + 5x^q$, where p and q are constants. **(2 marks)** Answer

(b) Given that $y = 3x^2 + 1 - \frac{2 + 5\sqrt{x}}{x}$, find $\frac{dy}{dx}$ **(4 marks)** Answer