Summary of key points

1 If $\overrightarrow{PQ} = \overrightarrow{RS}$ then the line segments PQ and RS are equal in length and are parallel.

2 $\overrightarrow{AB} = -\overrightarrow{BA}$ as the line segment AB is equal in length, parallel and in the opposite direction to BA.

3 **Triangle law for vector addition:** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

If $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$, then $\mathbf{a} + \mathbf{b} = \mathbf{c}$

4 Subtracting a vector is equivalent to 'adding a negative vector': $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

5 Adding the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ gives the zero vector $\mathbf{0}$: $\overrightarrow{PQ} + \overrightarrow{QP} = \mathbf{0}$.

6 Any vector parallel to the vector $\mathbf{a}$ may be written as $\lambda\mathbf{a}$, where λ is a non-zero scalar.

7 To multiply a column vector by a scalar, multiply each component by the scalar: $\lambda\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \lambda p \\ \lambda q \end{pmatrix}$

8 To add two column vectors, add the x-components and the y-components $\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$

9 A unit vector is a vector of length 1. The unit vectors along the x- and y-axes are usually denoted by $\mathbf{i}$ and $\mathbf{j}$ respectively. $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

10 For any two-dimensional vector: $\begin{pmatrix} p \\ q \end{pmatrix} = p\mathbf{i} + q\mathbf{j}$

11 For the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} = \begin{pmatrix} x \\ y \end{pmatrix}$, the magnitude of the vector is given by: $|\mathbf{a}| = \sqrt{x^2 + y^2}$

12 A unit vector in the direction of $\mathbf{a}$ is $\dfrac{\mathbf{a}}{|\mathbf{a}|}$

13 In general, a point P with coordinates (p, q) has position vector:

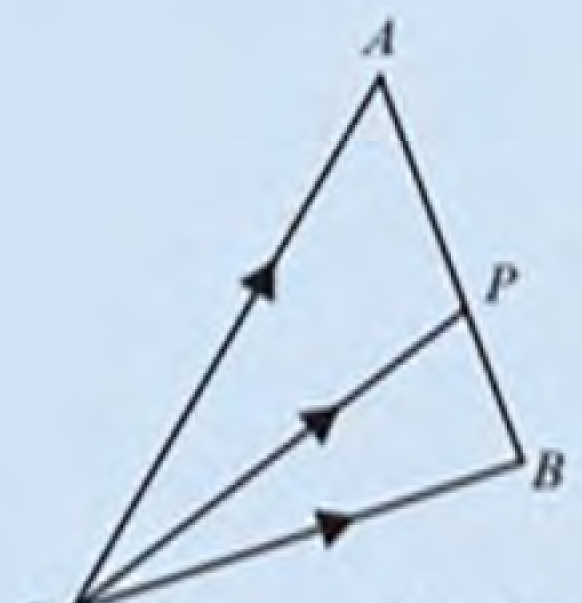
$$\overrightarrow{OP} = p\mathbf{i} + q\mathbf{j} = \begin{pmatrix} p \\ q \end{pmatrix}$$

14 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the position vectors of A and B respectively.

15 If the point P divides the line segment AB in the ratio $\lambda : \mu$, then

$$\overrightarrow{OP} = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}\overrightarrow{AB}$$

$$= \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}(\overrightarrow{OB} - \overrightarrow{OA})$$



$AP : PB = \lambda : \mu$

16 If $\mathbf{a}$ and $\mathbf{b}$ are two non-parallel vectors and $p\mathbf{a} + q\mathbf{b} = r\mathbf{a} + s\mathbf{b}$ then $p = r$ and $q = s$

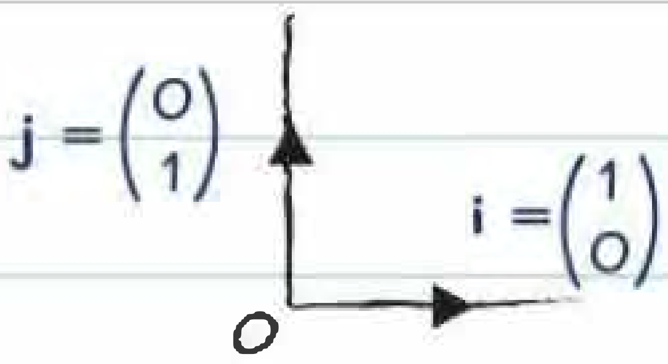
Had a look ☐ Nearly there ☐ Nailed it! ☐

Vectors

Vectors can be described using column vectors, or using **i**, **j** notation:

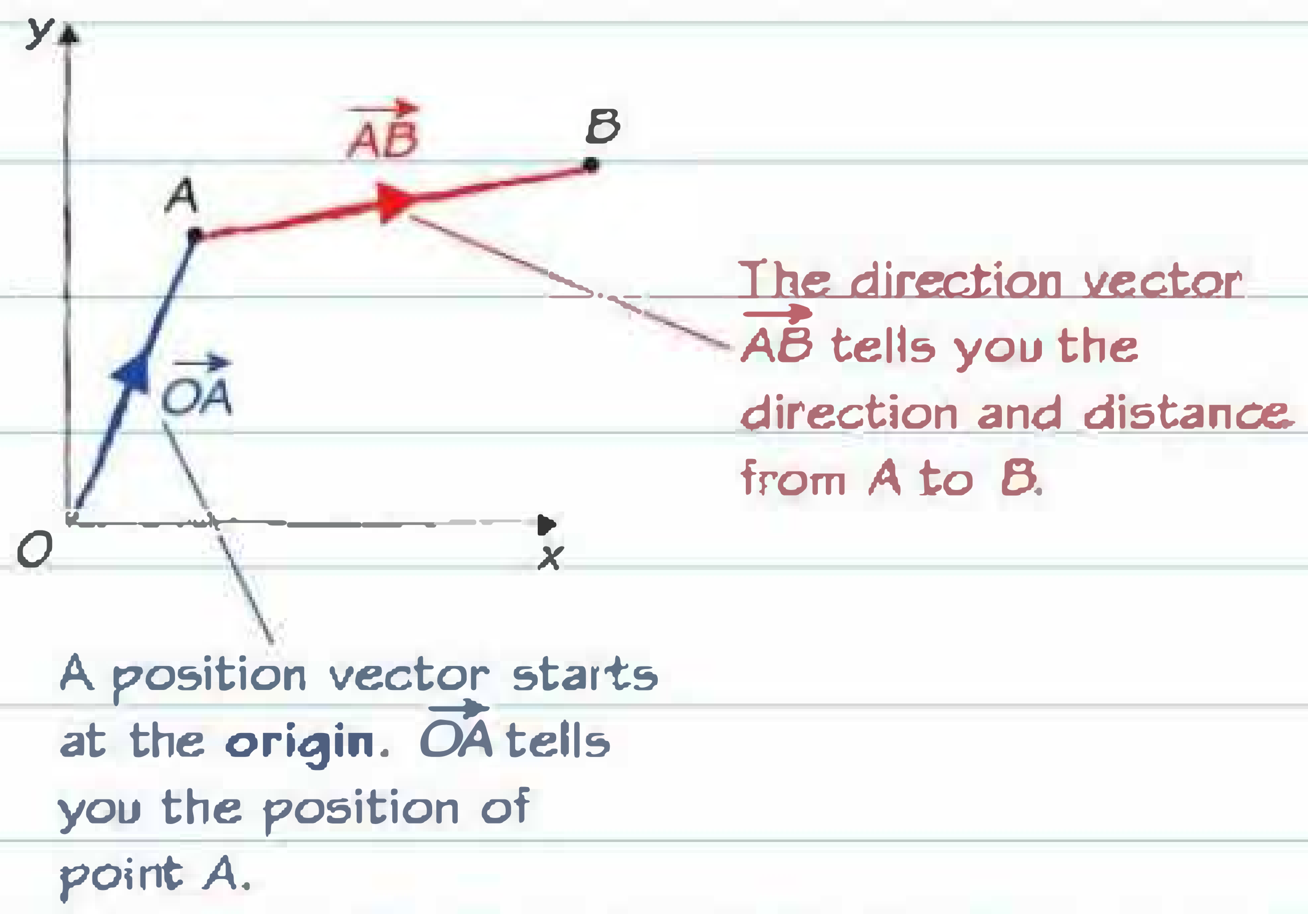
$$\overrightarrow{XY} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} = 3\mathbf{i} - \mathbf{j}$$

i and **j** are **perpendicular unit vectors**.



Position or direction?

It is useful to distinguish between **position vectors** and **direction vectors**.



Magnitude

You can find the magnitude of a vector using Pythagoras' theorem.

$$|\overrightarrow{AB}| = \left|\begin{pmatrix} 2 \\ -4 \end{pmatrix}\right| = |2\mathbf{i} - 4\mathbf{j}|$$

$$= \sqrt{2^2 + 4^2} = 2\sqrt{5}$$

Ignore minus signs when calculating the magnitude of a vector.

✓ **unit vectors** have magnitude 1.

✓ The **distance** between two points A and B is the magnitude of the vector $\overrightarrow{AB}$.

Worked example

The points P and Q have position vectors $3\mathbf{i} + 4\mathbf{j}$ and $-\mathbf{i} + 5\mathbf{j}$ respectively.

(a) Find the vector $\overrightarrow{PQ}$. **(2 marks)**

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$= (-1 - 3)\mathbf{i} + (5 - 4)\mathbf{j}$

$= -4\mathbf{i} + \mathbf{j}$

(b) Find the distance PQ. **(1 mark)**

$|\overrightarrow{PQ}| = \sqrt{-4^2 + 1^2}$

$= \sqrt{17}$

(c) Find a unit vector in the direction of $\overrightarrow{PQ}$. **(1 mark)**

$\frac{1}{\sqrt{17}}\overrightarrow{PQ} = \frac{1}{\sqrt{17}}(-4\mathbf{i} + \mathbf{j})$

$= -\frac{4}{\sqrt{17}}\mathbf{i} + \frac{1}{\sqrt{17}}\mathbf{j}$

$\overrightarrow{PQ} = \begin{pmatrix}\text{Position}\\\text{vector of } Q\end{pmatrix} - \begin{pmatrix}\text{Position}\\\text{vector of } P\end{pmatrix}$

You could also use column vectors to subtract:

$$\begin{pmatrix} -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -1-3 \\ 5-4 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$$

You can't write in bold in your exam! You can underline vectors to make them clearer. If you're writing the vector between two points, you should draw an arrow over the top. $\overrightarrow{PQ}$ is the **direction vector** from P to Q, whereas PQ is the **line segment** between P and Q.

Now try this

1 The points A and B have position vectors $4\mathbf{i} - 2\mathbf{j}$ and $5\mathbf{i} + 2\mathbf{j}$ respectively.

(a) Find the vector $\overrightarrow{AB}$. **(2 marks)** Answer

(b) Write down the vector $\overrightarrow{BA}$. **(1 mark)** Answer

2 Find a unit vector in the direction of $\begin{pmatrix} 2 \\ -10 \end{pmatrix}$ **(2 marks)** Answer