Summary of key points

1 A **probability distribution** fully describes the probability of any outcome in the sample space.

2 The sum of the probabilities of all outcomes of an event add up to 1. For a random variable X, you can write $\sum P(X = x) = 1$ for all x.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Random variables

A **discrete random variable** can take a range of discrete **numerical values**.
To define a random variable you need to know the range of values it can take (its **sample space**) and the probability that it takes each one. The probability that the random variable takes a certain value is given by its **probability function**.

The sum of probabilities

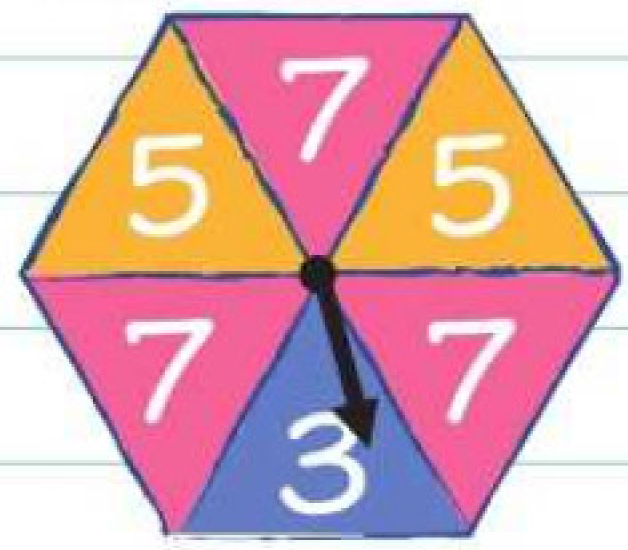
The most important fact you will use about random variables in your exam is that the probabilities of all the possible values of any random variable, X, always add up to 1.
Another way of saying this is:

$$\sum_{\text{all } x} P(X = x) = 1$$

You always use **upper case** letters for random variables, and **lower case** letters for the values they can take. $P(X = x)$ means 'the probability that the random variable X takes the value x'.

Probability distributions

You can write the outcome from this spinner as a random variable X.



You can write its **probability distribution** in a table.

x	3	5	7
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

This is the **sample space** for this random variable. X can only take these values.

$\frac{1}{6} + \frac{1}{3} + \frac{1}{2} = 1$

Its probability distribution could also be given using a **probability function**:

$P(X = x) = \frac{x - 1}{12}, \quad x = 3, 5, 7$

For example $P(X = 5) = \frac{5 - 1}{12} = \frac{4}{12} = \frac{1}{3}$

Worked example

The random variable X has probability function

$$P(X = x) = \begin{cases} kx^2, & x = 1, 2, 3 \\ 2kx, & x = 4, 5 \end{cases}$$

where k is a constant.

(a) Find the value of k. **(2 marks)**

$k + 4k + 9k + 8k + 10k = 1$

$32k = 1$

$k = \frac{1}{32}$

(b) Find $P(X > 2)$. **(2 marks)**

$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5)$

$= \frac{9}{32} + \frac{8}{32} + \frac{10}{32} = \frac{27}{32}$

The curly bracket means that the probability function is different for different values of x. For $x = 1$, 2 and 3 you use $P(X = x) = kx^2$ and for $x = 4$ and 5 you use $P(X = x) = 2kx$.

For part (a), write an expression for the sum of the probabilities in terms of k. $\sum P(X = x) = 1$, so you can write an equation and solve it to find k.

For part (b), add up the probabilities for the values of X which make the inequality true.

Now try this

1 The random variable Y has probability function

$$P(Y = y) = \frac{(y - 1)^2}{30}, \quad y = 2, 3, 4, 5$$

(a) Construct a table giving the probability distribution of Y. Answer **(3 marks)**

(b) Find $P(Y > 3)$. Answer **(2 marks)**

For part (b), solve the inequality first.

2 The discrete random variable X has probability distribution given by

x	−2	−1	0	1	2
$P(X = x)$	0.1	a	0.15	$2a$	0.15

where a is a constant. Answer

(a) Find the value of a. **(2 marks)**

(b) Find $P(3X + 1 \leqslant 6)$. **(2 marks)** Answer