Summary of key points

1 A **probability distribution** fully describes the probability of any outcome in the sample space.

2 The sum of the probabilities of all outcomes of an event add up to 1. For a random variable X, you can write $\sum P(X = x) = 1$ for all x.

3 You can model X with a **binomial distribution**, **B(n, p)**, if:

- there are a fixed number of trials, n
- there are two possible outcomes (success or failure)
- there is a fixed probabillity of success, p
- the trials are independent of each other.

4 If a random variable X has the binomial distribution $B(n, p)$ then its probability mass function is given by

$$P(X = r) = \binom{n}{r} p^r (1 - p)^{n - r}$$

The binomial distribution

If you are carrying out a large number of trials you can model the number of **successful trials**, X, using a binomial distribution. For n trials, each with probability of success, p, you write:

$X \sim B(n, p)$

The probability that X takes a given value r is:

$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$

To bi or not to bi?

A binomial model is valid when

- ✓ there are a fixed number of trials
- ✓ the trials are independent
- ✓ there are two possible outcomes, with probabilities p and $1 - p$
- ✓ the probability of each outcome is fixed.

Worked example

The discrete random variable $X \sim B(35, 0.82)$. Find:

(a) $P(X = 29)$

$P(X = 29) = \binom{35}{29} 0.82^{29} 0.18^{6}$

$= 0.175$ (3 s.f.)

(b) $P(X \geqslant 25)$

$P(X \geqslant 25) = 1 - P(X \leqslant 24)$

$= 1 - 0.03877...$

$= 0.961$ (3 s.f.)

The easiest way to find binomial probabilities is using the binomial functions on your calculator. To find the probability that X takes a **single value** use the "Binomial probability distribution" function. You can also use the formula for $P(X = r)$ and the nCr function on your calculator to find a single binomial probability.

Binomial PD
x :29
N :35
p :0.82

To find the probability that X is **less than or equal to** a given value, use the "Binomial cumulative distribution" function. The question asks for $P(X \geqslant 25)$, so use the fact that the sum of the probabilities is equal to 1. To find $P(X \leqslant 24)$ type in:

Binomial CD
x :24
N :35
p :0.82

X can only take whole numbered values, so $P(X < 4) = P(X \leqslant 3)$.

Worked example

A chicken farmer claims that 7% of his eggs have double yolks. The farmer takes a random sample of 20 eggs.

(a) Find the probability that fewer than 4 of them have double yolks. **(2 marks)**

X = the number of double yolks in sample

$X \sim B(20, 0.07)$

$P(X \leqslant 3) = 0.953$ (3 s.f.)

(b) Give a reason why it would not be appropriate for the farmer to test her claim using a census. **(1 mark)**

If she tested all the eggs she would have none left to sell.

Now try this

1 A fair six-sided dice is rolled 50 times. The discrete random variable X represents the number of 1s rolled.

(a) Justify the use of a binomial distribution to model X. Answer **(2 marks)**

(b) Find (i) $P(X = 10)$ (ii) $P(X < 7)$ **(3 marks)**

Answer Answer

2 A bag contains 60 blue counters and 40 red counters. Emma selects 10 counters at random from the bag without replacement. She models the number of blue counters selected as B(10, 0.6). Explain why this is not a suitable model for this situation. Answer **(2 marks)**