Summary of key points

1. The null hypothesis, H_0, is the hypothesis that you assume to be correct.
2. The alternative hypothesis, H_1, tells us about the parameter if your assumption is shown to be wrong.
3. Hypothesis tests with alternative hypotheses in the form $H_1: p < \ldots$ and $H_1: p > \ldots$ are called one-tailed tests.
4. Hypothesis tests with an alternative hypothesis in the form $H_1: p \neq \ldots$ are called two-tailed tests.
5. A critical region is a region of the probability distribution which, if the test statistic falls within it, would cause you to reject the null hypothesis.
6. The critical value is the first value to fall inside of the critical region.
7. The actual significance level of a hypothesis test is the probability of incorrectly rejecting the null hypothesis.
8. For a two-tailed test the critical region is split at either end of the distribution.
9. For a two-tailed test, either double the *p*-value for your observation, or halve the significance level at the end you are testing.

Hypothesis testing

You need to be able to carry out a hypothesis test for the probability, p, in a binomial distribution. Follow these steps to carry out a hypothesis test.

Model the test statistic and define null (H_0) and alternative (H_1) hypotheses. → Assume H_0 is true and calculate the probabilityof the observed value (or a greater / lesser value) occurring → Compare this probability with a given significance level and write a conclusion stating whether H_0 is accepted or rejected.

Worked example

A microchip manufacturer knows that 9% of the microchips produced using a certain process contain defects. The manufacturer trials a new manufacturing process. A sample of 50 chips from the new process are selected and 2 of them are observed to be faulty.

Test, at the 10% significance level, whether there is evidence that the proportion of faulty chips has reduced under the new process. State your hypotheses clearly. **(6 marks)**

Let X = the number of faulty chips in a sample of 50. Then $X \sim B(50, p)$.

$H_0: p = 0.09$, $H_1: p < 0.09$

Assume H_0 is true, so $X \sim B(50, 0.09)$.

$P(X \leqslant 2) = 0.1605...$

16% > 10% so there is not enough evidence to reject H_0.

The proportion of faulty chips has not significantly reduced under the new process.

How many tails?

- If you want to test whether p is likely to be **greater than** or **less than** a particular value you need to use a one-tailed test. For example:
  $H_0: p = 0.4$, $H_1: p > 0.4$
- If you want to test whether p is likely to be **different** from a particular value, you need to use a two-tailed test. For example:
  $H_0: p = 0.75$, $H_1: p \neq 0.75$

Problem solved!

You want to test whether the proportion has **reduced** so this is a **one-tailed test**.

You can use your calculator to find $P(X \leqslant 2)$ directly. Since the probability of this observation (or worse) is **greater than** 10% you **do not** reject H_0.

You will need to use problem-solving skills throughout your exam – **be prepared!**

Now try this

1 A random variable has distribution $X \sim B(28, p)$. A single observation of $x = 25$ is taken from this distribution. Test, at the 2% significance level, $H_0: p = 0.7$ against $H_1: p > 0.7$. Answer **(5 marks)**

2 Between April 2015 and March 2016, the national average pass rate for the car driving theory exam was 49.3%. From the same period, a sample of 45 students in Edinburgh was taken, and 29 of them were found to have passed the exam.

Test, at the 5% significance level, whether the average pass rate in Edinburgh was different from the national average pass rate. You must state your hypotheses clearly. Answer **(6 marks)**

This is a **two-tailed test** so you need to **halve** the significance level at each end of the test. The national pass rate is 49.3% so you would expect about 22 students to pass. Calculate $P(X \geqslant 29)$ and compare the result with 2.5%.

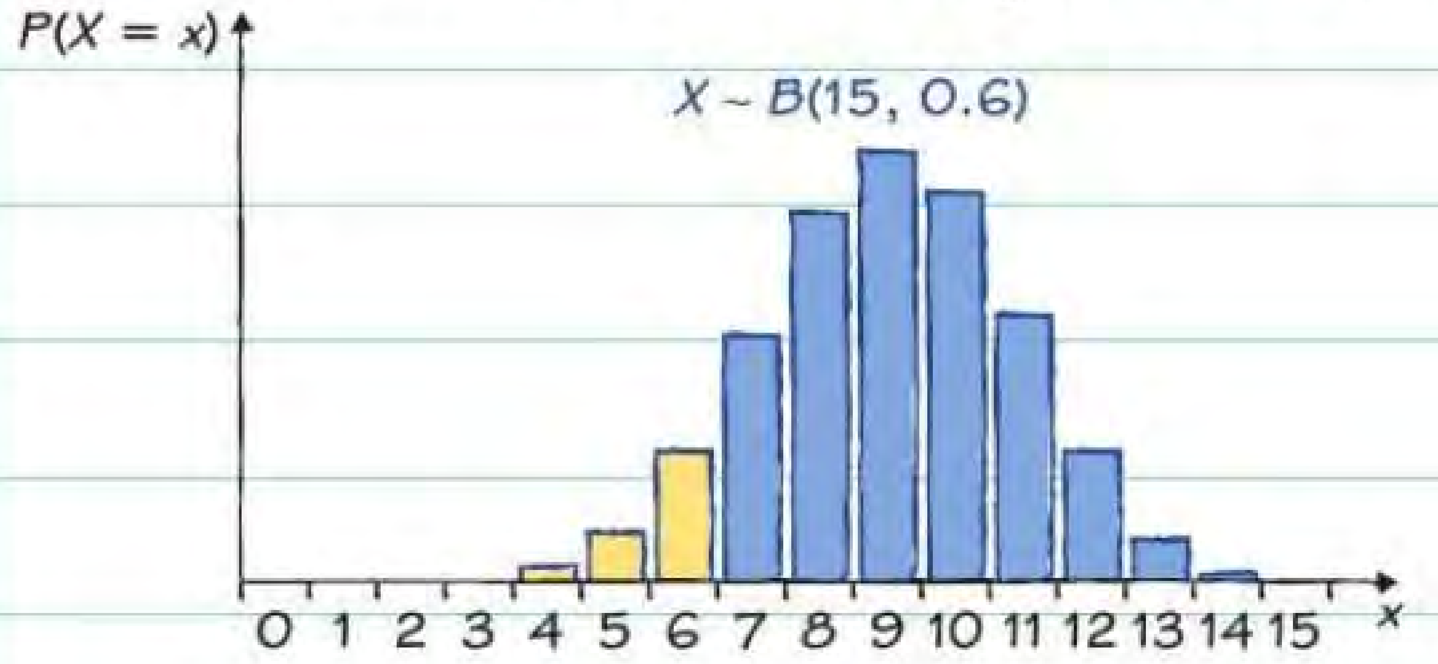
Had a look ☐ Nearly there ☐ Nailed it! ☐

Critical regions

In a hypothesis test, you can find the observed values of a random variable which would cause you to reject the null hypothesis. This set of values is called the **critical region**.

1 One-tailed tests

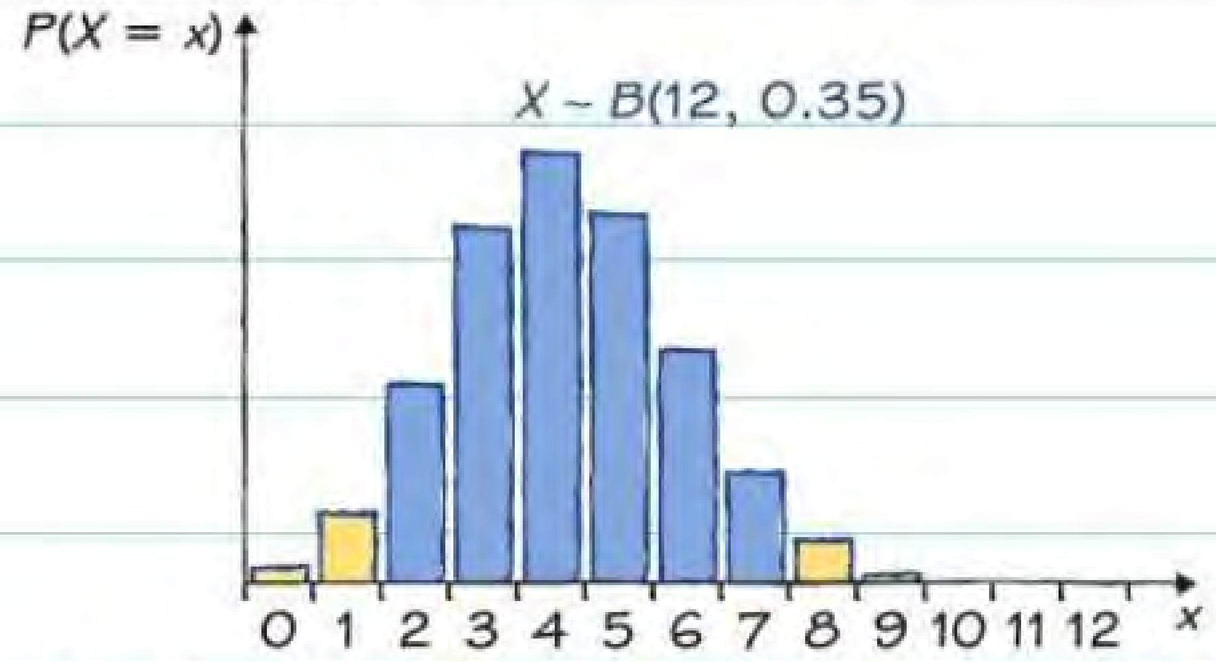
For $X \sim B(15, p)$, if you are testing H_0: $p = 0.6$ against H_1: $p < 0.6$ at the 10% level, the critical region is $X \leqslant 6$.



$P(X \leqslant 6) = 0.09505$, which is the first value of x with $P(X \leqslant x) < 0.1$. The value 6 is called the **critical value**.

2 Two-tailed tests

For $X \sim B(12, p)$, if you are testing H_0: $p = 0.35$ against H_1: $p \neq 0.35$ at the 10% level, the critical region is $X \leqslant 1$ and $X \geqslant 8$.



Divide the significance level by 2. The probability in **each tail** should be $\leqslant 0.05$. $P(X \leqslant 1) = 0.04244$ and $P(X \geqslant 8) = 0.02551$. There are two **critical values**: 1 and 8.

Worked example

The random variable X is modelled as $X \sim B(20, p)$. A single observation of X is taken and used to test H_0: $p = 0.3$ against H_1: $p > 0.3$.

(a) Using a 5% significance level, find the critical region for this test. **(2 marks)**

Assume H_0 is true, so $X \sim B(20, 0.3)$

$P(X \geqslant 9) = 1 - P(X \geqslant 8) = 1 - 0.8867 = 0.1133$

$P(X \geqslant 10) = 1 - P(X \leqslant 9) = 1 - 0.9520 = 0.0480$

The critical region is $X \geqslant 10$

(b) State the actual significance level of this test. **(1 mark)**

0.0480

The actual observed value of X is 11.

(c) Comment on this observation in light of your critical region. **(2 marks)**

11 lies inside the critical region, so this is evidence to reject H_0 at the 5% significance level.

Actual significance level

The actual probability that the observed value will fall within the critical region is sometimes called the actual significance level. This is also the probability that the null hypothesis is **rejected incorrectly**.

You are testing whether $p > 0.3$, so the critical value is the first value x such that $P(X \geqslant x) < 0.05$, or $P(X \geqslant x - 1) > 0.95$. You can use the **binomial cumulative distribution** table in the formulae booklet:

$p =$	0.25	0.30	0.35
$n = 20, x = 7$	0.8982	0.7723	0.6010
8	0.9591	0.8867	0.7624
9	0.9861	0.9520	0.8782
10	0.9961	0.9829	0.9468

Watch out! In part (a) you need to find probabilities **as close as possible** to 0.01. This means that the probabilities in each tail do not necessarily have to be less than 0.01.

Now try this

A single observation is taken from a binomial distribution $B(15, p)$, and is used to test H_0: $p = 0.45$ against H_1: $p \neq 0.45$

(a) Using a 2% level of significance, find the critical region for this test. The probability in each tail should be as close to 0.01 as possible. Answer **(3 marks)**

(b) Find the actual significance level of this test. Answer **(2 marks)**