## Summary of key points

**1** If the general term, $u_r$, of a series can be expressed in the form $f(r) - f(r+1)$

then $\sum_{r=1}^{n} u_r = \sum_{r=1}^{n} (f(r) - f(r+1))$.

so
$$u_1 = f(1) - f(2)$$
$$u_2 = f(2) - f(3)$$
$$u_3 = f(3) - f(4)$$
$$\vdots$$
$$u_n = f(n) - f(n+1)$$

Then adding $\sum_{r=1}^{n} u_r = f(1) - f(n+1)$

**2** The **Maclaurin series** of a function $f(x)$ is given by

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(r)}(0)}{r!}x^r + \ldots$$

The series is valid provided that $f(0), f'(0), f''(0), \ldots, f^{(r)}(0), \ldots$ all have finite values.

**3** The following Maclaurin series are given in the formulae booklet:

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad -1 < x \leqslant 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad -1 \leqslant x \leqslant 1$$